

TABLES FOR STEPWISE MULTIPLE COMPARISON PROCEDURES

by

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ABSTRACT

This paper presents tables for $\alpha = .05$ and degrees of freedom equal to 5(1)20, 24, 30, 40, 60, 120, and ∞ for the multiple comparison procedures discussed in the article "Stepwise Multiple Comparison Procedures" by Roy E. Welsch in the September 1977 issue of the Journal of the American Statistical Association.

USE OF THE TABLES

First, find the column with the total number of means (treatments) to be compared, say 5. For 20 degrees of freedom the GAPA (step-up) critical numbers would be 3.58, 3.97, 3.97, and 4.29 for gaps, 3-stretches, 4-stretches, and the range. When the GAPB numbers differ from those for GAPA, they are written in parentheses below the GAPA numbers. Thus for 20 degrees of freedom the GAPB critical numbers are 3.58, 3.97, 4.13, and 4.24.

The same rules apply for the step-down tests, NKA and NKB. Again for 5 means and 20 degrees of freedom the critical numbers for NKA are 4.23, 3.96, 3.93, and 3.58 for the range, 4-stretch, 3-stretch, and gaps. For NKB they are 4.23, 4.11, 3.93, and 3.58.

Linear harmonic interpolation is recommended for other degrees of freedom. All tables are for the .05 significance level. For further details see the author's article "Stepwise Multiple Comparison Procedures" in the September 1977 issue of the Journal of the American Statistical Association.

NKA(B) Critical Numbers

DF = 5

Stretch	Number of Means								
	<u>10</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>
10	6.99								
9	6.97 (6.99)	6.80							
8	6.97	6.75 (6.78)	6.58						
7	6.93	6.75	6.50 (6.55)	6.33					
6	6.87	6.69	6.50	6.19 (6.28)	6.03				
5	6.78	6.60	6.41	6.19	5.81 (5.95)	5.67			
4	6.61	6.44	6.26	6.05	5.81	5.30 (5.54)	5.22		
3	6.32	6.16	5.98	5.78	5.56	5.30	4.60 (4.99)	4.60	
2	5.70	5.55	5.39	5.21	5.00	4.76	4.47	3.64 (4.12)	3.64

DF = 6

10	6.49								
9	6.44 (6.47)	6.32							
8	6.44	6.25 (6.29)	6.12						
7	6.40	6.25	6.02 (6.08)	5.90					
6	6.33	6.18	6.02	5.74 (5.84)	5.63				
5	6.23	6.08	5.92	5.74	5.40 (5.54)	5.31			
4	6.07	5.93	5.77	5.60	5.40	4.94 (5.17)	4.90		
3	5.80	5.66	5.51	5.35	5.16	4.94	4.34 (4.67)	4.34	
2	5.24	5.12	4.98	4.83	4.65	4.44	4.20	3.46 (3.89)	3.46

NKA(B) Critical Numbers

DF = 7

Stretch	Number of Means								
	<u>10</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>
10	6.16								
9	6.09 (6.13)	6.00							
8	6.09	5.91 (5.96)	5.82						
7	6.04	5.91	5.70 (5.77)	5.61					
6	5.97	5.84	5.70	5.45 (5.54)	5.36				
5	5.87	5.74	5.60	5.45	5.13 (5.27)	5.06			
4	5.71	5.59	5.46	5.30	5.13	4.70 (4.93)	4.68		
3	5.46	5.34	5.21	5.06	4.90	4.70	4.17 (4.47)	4.17	
2	4.95	4.84	4.72	4.58	4.42	4.24	4.02	3.34 (3.74)	3.34

DF = 8

10	5.92								
9	5.85 (5.89)	5.77							
8	5.85	5.68 (5.73)	5.60						
7	5.79	5.68	5.48 (5.55)	5.40					
6	5.72	5.60	5.48	5.24 (5.33)	5.17				
5	5.62	5.50	5.38	5.24	4.94 (5.08)	4.89			
4	5.46	5.36	5.23	5.10	4.94	4.54 (4.75)	4.53		
3	5.22	5.12	5.00	4.87	4.71	4.54	4.04 (4.32)	4.04	
2	4.75	4.65	4.53	4.41	4.27	4.10	3.89	3.26 (3.63)	3.26

NKA(B) Critical Numbers

DF = 9

Stretch	Number of Means								
	<u>10</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>
10	5.74								
9	5.66 (5.71)	5.60							
8	5.66	5.50 (5.55)	5.43						
7	5.61	5.50	5.31 (5.38)	5.24					
6	5.53	5.43	5.31	5.08 (5.18)	5.02				
5	5.43	5.33	5.21	5.08	4.80 (4.93)	4.76			
4	5.28	5.18	5.07	4.94	4.80	4.41 (4.62)	4.41		
3	5.05	4.95	4.84	4.72	4.58	4.41	3.95 (4.21)	3.95	
2	4.60	4.50	4.40	4.28	4.15	3.99	3.80	3.20 (3.55)	3.20

DF = 10

10	5.60								
9	5.52 (5.56)	5.46							
8	5.52	5.36 (5.42)	5.30						
7	5.46	5.36	5.18 (5.25)	5.12					
6	5.39	5.29	5.18	4.96 (5.06)	4.91				
5	5.29	5.19	5.08	4.96	4.69 (4.82)	4.65			
4	5.14	5.05	4.94	4.82	4.69	4.33 (4.53)	4.33		
3	4.92	4.82	4.72	4.61	4.47	4.32	3.88 (4.13)	3.88	
2	4.48	4.39	4.30	4.19	4.06	3.91	3.73	3.15 (3.49)	3.15

NKA(B) Critical Numbers

DF = 11

Stretch	Number of Means								
	<u>10</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>
10	5.49								
9	5.40 (5.45)	5.35							
8	5.40	5.25 (5.31)	5.20						
7	5.35	5.25	5.08 (5.15)	5.03					
6	5.27	5.18	5.08	4.86 (4.96)	4.82				
5	5.17	5.08	4.98	4.86	4.60 (4.73)	4.57			
4	5.03	4.94	4.84	4.73	4.60	4.26 (4.45)	4.26		
3	4.81	4.72	4.63	4.52	4.39	4.24	3.82 (4.06)	3.82	
2	4.39	4.31	4.22	4.11	3.99	3.84	3.67	3.11 (3.44)	3.11

DF = 12

10	5.40								
9	5.31 (5.36)	5.27							
8	5.31	5.16 (5.22)	5.12						
7	5.25	5.16	4.99 (5.06)	4.95					
6	5.18	5.09	4.99	4.79 (4.88)	4.75				
5	5.08	4.99	4.90	4.79	4.53 (4.66)	4.51			
4	4.94	4.86	4.76	4.65	4.53	4.20 (4.38)	4.20		
3	4.73	4.64	4.55	4.45	4.32	4.18	3.77 (4.00)	3.77	
2	4.32	4.24	4.15	4.05	3.93	3.79	3.62	3.08 (3.40)	3.08

NKA(B) Critical Numbers

DF = 13

Stretch	Number of Means								
	<u>10</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>
10	5.32								
9	5.23 (5.28)	5.19							
8	5.23	5.09 (5.15)	5.05						
7	5.18	5.09	4.92 (4.99)	4.88					
6	5.10	5.02	4.92	4.72 (4.81)	4.69				
5	5.00	4.92	4.83	4.72	4.47 (4.60)	4.45			
4	4.87	4.79	4.69	4.59	4.47	4.15 (4.33)	4.15		
3	4.66	4.58	4.49	4.39	4.27	4.13	3.73 (3.96)	3.73	
2	4.26	4.18	4.10	4.00	3.88	3.75	3.58	3.06 (3.37)	3.06

DF = 14

10	5.25								
9	5.17 (5.22)	5.13							
8	5.17	5.03 (5.08)	4.99						
7	5.11	5.03	4.86 (4.94)	4.83					
6	5.04	4.96	4.86	4.67 (4.76)	4.64				
5	4.94	4.86	4.77	4.67	4.42 (4.55)	4.41			
4	4.80	4.73	4.64	4.54	4.42	4.11 (4.28)	4.11		
3	4.60	4.52	4.43	4.34	4.22	4.09	3.70 (3.92)	3.70	
2	4.21	4.14	4.05	3.95	3.84	3.71	3.55	3.03 (3.34)	3.03

NKA(B) Critical Numbers

DF = 15

Stretch	Number of Means								
	<u>10</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>
10	5.20								
9	5.11 (5.16)	5.08							
8	5.11	4.97 (5.03)	4.94						
7	5.05	4.97	4.81 (4.88)	4.78					
6	4.98	4.90	4.81	4.62 (4.71)	4.59				
5	4.89	4.81	4.72	4.62	4.38 (4.51)	4.37			
4	4.75	4.68	4.59	4.49	4.38	4.08 (4.24)	4.08		
3	4.55	4.47	4.39	4.29	4.18	4.05	3.67 (3.89)	3.67	
2	4.17	4.09	4.01	3.92	3.81	3.68	3.52	3.01 (3.31)	3.01

DF = 16

10	5.15								
9	5.06 (5.11)	5.03							
8	5.06	4.93 (4.99)	4.90						
7	5.01	4.93	4.77 (4.84)	4.74					
6	4.93	4.86	4.77	4.58 (4.67)	4.56				
5	4.84	4.76	4.68	4.58	4.34 (4.47)	4.33			
4	4.71	4.63	4.55	4.45	4.34	4.05 (4.21)	4.05		
3	4.51	4.43	4.35	4.26	4.15	4.02	3.65 (3.86)	3.65	
2	4.13	4.06	3.98	3.89	3.78	3.65	3.50	3.00 (3.29)	3.00

NKA(B) Critical Numbers

DF = 17

Stretch	Number of Means								
	<u>10</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>
10	5.11								
9	5.02 (5.07)	4.99							
8	5.02	4.89 (4.95)	4.86						
7	4.96	4.89	4.73 (4.80)	4.71					
6	4.89	4.82	4.73	4.55 (4.64)	4.52				
5	4.80	4.72	4.64	4.55	4.31 (4.43)	4.30			
4	4.67	4.59	4.51	4.42	4.31	4.02 (4.18)	4.02		
3	4.47	4.40	4.32	4.22	4.12	3.99	3.63 (3.83)	3.63	
2	4.10	4.03	3.95	3.86	3.76	3.63	3.48	2.98 (3.28)	2.98

DF = 18

10	5.07								
9	4.98 (5.03)	4.96							
8	4.98	4.85 (4.91)	4.82						
7	4.93	4.85	4.70 (4.77)	4.67					
6	4.85	4.78	4.70	4.51 (4.60)	4.49				
5	4.76	4.69	4.61	4.51	4.28 (4.41)	4.28			
4	4.63	4.56	4.48	4.39	4.28	4.00 (4.16)	4.00		
3	4.44	4.37	4.29	4.20	4.09	3.97	3.61 (3.81)	3.61	
2	4.07	4.00	3.92	3.84	3.73	3.61	3.46	2.97 (3.26)	2.97

NKA(B) Critical Numbers

DF = 24

Stretch	Number of Means								
	<u>10</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>
10	4.92								
9	4.83 (4.88)	4.81							
8	4.83	4.70 (4.76)	4.68						
7	4.77	4.70	4.56 (4.63)	4.54					
6	4.70	4.63	4.56	4.38 (4.47)	4.37				
5	4.61	4.54	4.47	4.38	4.17 (4.29)	4.17			
4	4.49	4.42	4.35	4.26	4.17	3.90 (4.05)	3.90		
3	4.30	4.24	4.16	4.08	3.98	3.87	3.53 (3.72)	3.53	
2	3.96	3.89	3.82	3.74	3.64	3.52	3.38	2.92 (3.19)	2.92

DF = 30

10	4.82								
9	4.74 (4.78)	4.72							
8	4.74	4.62 (4.67)	4.60						
7	4.68	4.62	4.48 (4.55)	4.46					
6	4.61	4.55	4.48	4.31 (4.40)	4.30				
5	4.52	4.46	4.39	4.31	4.10 (4.22)	4.10			
4	4.40	4.34	4.27	4.19	4.10	3.85 (3.99)	3.85		
3	4.22	4.16	4.09	4.01	3.92	3.81	3.49 (3.67)	3.49	
2	3.89	3.83	3.76	3.68	3.59	3.48	3.34	2.89 (3.15)	2.89

NKA(B) Critical Numbers

DF = 19

Stretch	Number of Means									
	<u>10</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>	
10	5.04									
9	4.95 (5.00)	4.92								
8	4.95	4.82 (4.88)	4.79							
7	4.89	4.82	4.67 (4.74)	4.65						
6	4.82	4.75	4.67	4.49 (4.58)	4.47					
5	4.73	4.66	4.58	4.49	4.26 (4.38)	4.25				
4	4.60	4.53	4.45	4.36	4.26	3.98 (4.13)	3.98			
3	4.41	4.34	4.26	4.17	4.07	3.95	3.59 (3.79)	3.59		
2	4.05	3.98	3.90	3.81	3.71	3.59	3.44	2.96 (3.24)	2.96	

DF = 20

10	5.01									
9	4.92 (4.97)	4.90								
8	4.92	4.79 (4.85)	4.77							
7	4.86	4.79	4.64 (4.71)	4.62						
6	4.79	4.72	4.64	4.46 (4.55)	4.45					
5	4.70	4.63	4.55	4.46	4.23 (4.36)	4.23				
4	4.57	4.50	4.43	4.34	4.23	3.96 (4.11)	3.96			
3	4.38	4.31	4.24	4.15	4.05	3.93	3.58 (3.78)	3.58		
2	4.02	3.96	3.88	3.80	3.70	3.58	3.43	2.95 (3.23)	2.95	

NKA(B) Critical Numbers

DF = 120

Stretch	Number of Means								
	<u>10</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>
10	4.56								
9	4.48 (4.52)	4.47							
8	4.48	4.37 (4.42)	4.36						
7	4.42	4.37	4.25 (4.31)	4.24					
6	4.36	4.31	4.25	4.10 (4.18)	4.10				
5	4.28	4.22	4.16	4.10	3.92 (4.01)	3.92			
4	4.17	4.11	4.05	3.98	3.90	3.68 (3.81)	3.68		
3	4.00	3.95	3.89	3.82	3.74	3.64	3.36 (3.52)	3.36	
2	3.70	3.65	3.59	3.52	3.43	3.33	3.21	2.80 (3.04)	2.80

DF = ∞

10	4.47								
9	4.39 (4.44)	4.39							
8	4.39	4.29 (4.34)	4.29						
7	4.34	4.29	4.17 (4.23)	4.17					
6	4.28	4.23	4.17	4.03 (4.11)	4.03				
5	4.20	4.15	4.09	4.03	3.86 (3.95)	3.86			
4	4.09	4.04	3.98	3.92	3.84	3.63 (3.75)	3.63		
3	3.93	3.88	3.83	3.76	3.68	3.59	3.31 (3.47)	3.31	
2	3.64	3.59	3.53	3.46	3.39	3.29	3.17	2.77 (3.01)	2.77

NKA(B) Critical Numbers

DF = 40

Stretch	Number of Means								
	<u>10</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>
10	4.73								
9	4.65 (4.70)	4.63							
8	4.65	4.53 (4.59)	4.52						
7	4.59	4.53	4.40 (4.47)	4.39					
6	4.53	4.47	4.40	4.24 (4.32)	4.23				
5	4.44	4.38	4.31	4.24	4.04 (4.15)	4.04			
4	4.32	4.26	4.20	4.12	4.03	3.79 (3.92)	3.79		
3	4.15	4.09	4.02	3.95	3.86	3.75	3.44 (3.62)	3.44	
2	3.82	3.77	3.70	3.62	3.53	3.43	3.29	2.86 (3.12)	2.86

DF = 60

10	4.65								
9	4.56 (4.61)	4.55							
8	4.56	4.45 (4.51)	4.44						
7	4.51	4.45	4.32 (4.39)	4.31					
6	4.44	4.38	4.32	4.17 (4.25)	4.16				
5	4.36	4.30	4.24	4.17	3.98 (4.08)	3.98			
4	4.24	4.19	4.12	4.05	3.97	3.74 (3.87)	3.74		
3	4.07	4.02	3.95	3.88	3.80	3.70	3.40 (3.57)	3.40	
2	3.76	3.71	3.64	3.57	3.48	3.38	3.25	2.83 (3.08)	2.83