Dynamic Ship Assignment Problem with Uncertain Demands

by

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Submitted to the Department of Civil and Environmental Engineering in partial fulfillment of the requirements for the degree of

Master of Science in Transportation

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

February 2016

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Abstract

Product tanker shipping companies in the spot market face severe volatility in demand and in price. We explore shipping companies' two problems: evaluating supply and demand of the market and assigning cargoes in order to maximize profitability. By approximating the market as a queueing system, we obtain utilization ratios, which effectively model supply and demand of the market. This approach directly evaluates the impact of ton-mile on utilization ratios, even a small growth of which may result in a significant supply shortage. Queueing approximation also allows formulation of dynamic ship assignment as a semi-Markov average cost problem, attaining stationary policies. When profit margins are low, a stationary policy frequently rejects cargoes. Despite rejections, it yields the highest profit per cargo, compared to other methods. Such optimal controls remain valid even when demand fluctuates.

Thesis Supervisor: Christopher G. Caplice
Title: Executive Director, Center for Transportation and Logistics

Acknowledgments

This study would not be possible without the help of the following persons: Dr. Chris Caplice, for his invaluable advices and insights, Jakob Tolstrup-Moller of Clarksons, for the provision of dataset that has opened the door for this study, Jung Hee Park, for his advices on search algorithms, Hun Lim, for his introduction to the industry for the novice, Peiguang Hu, for his collaboration on our game theory project, Professor Dimitri Bertsekas, for his teachings and advices on dynamic programming, and last but least, Hyo Young and Samuel, for everything.

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Chapter 1

Introduction

We introduce the product tanker market with a focus on the spot market. Transportation demand in the spot market is inherently uncertain and ships constantly face idling time and empty voyages. We also introduce the controller who allocates shipments and we address its problems: evaluating the utilization ratio of ships in a region and assigning cargoes[1] in order to maximize profitability. Finally, we observe in the market data that cargo arrivals in this market follow Poisson processes.

1.1 Product tanker market

Oil refineries produce various types of petroleum products, which are sold to buyers everywhere according to demand and market price. The distance between production and consumption drives the global trade of petroleum products, as exemplified by a large annual volume of naphtha trade from the Middle East to Northeastern Asia. Geographical arbitrage is another key driver. Traders create demand for ocean transportation when there is a large gap in the price of the same product between two regions. Naphtha, gas oil and fuel oil are the three major products subject to arbitrage trade. Of the 45 million barrels of petroleum products consumed per day globally, almost half (21 million) are traded by sea (Clarkson, 2014). Such seaborne trade is most commonly carried by handysize product tankers with 30,000-40,000 ton

[1]"Cargo", "load", and "shipload" are used interchangeably in this thesis.

cargo capacity, also known as medium range (MR) tankers. Product tankers provide so called pickup and delivery service whenever there is a demand (it is called loading and discharging in the maritime industry).

The product tanker market experiences severe volatility because transportation demand is driven by demand for different petroleum products, which is constantly changing in different regions. Due to such uncertainty, shippers (cargo owners) and shipping companies (carriers) usually do not commit to long-term contracts. Instead, a large number (exact figure unknown) of cargoes are transacted in the spot market based on loosely binding contracts for each cargo which can be canceled without penalty if the ship becomes unavailable. MR tankers are standardized, so petroleum products are moved by similar cargo size and at similar speed. Hence, a ship rarely has any competitive advantage over other ships and is a price taker who carries cargo at market price, which is highly volatile. The product tanker market is highly fragmented and players with differently sizes (mostly small) compete fiercely.

Since petroleum products are liquid and ships are mostly fully loaded with the cargo by a single shipper, cargo is differentiated only by the loading and discharging ports, pickup deadline and charter hire (price), not by types of cargoes. A ship's status is always defined as either laden (fully loaded) voyage or ballast (empty) voyage when sailing.

1.2 Controller

In addition to providing transportation services in the spot market, product tanker shipping companies also carry cargoes on long-term contracts (longer than a single contract) and charter out (lease) ships for a certain period to lock in earnings. This paper deals only with the spot market and we call a tanker shipping company in this context a "controller." The controller is the one who manages a fleet of ships and assigns each cargo to one of the available ships. We call each cargo-ship assignment decision "control." In the real market, a controller may be an individual shipowner, or a corporate entity that manages owned and chartered ships, or a pool operator who is consigned a number of ships from various shipowners and runs these ships as one pool. Typical large controllers have dozens of ships in their fleets (Lim, 2015).

1.3 Controller's Problem

Due to uncertainty in both demand and charter hire, controllers always try to evaluate supply and demand of the market. Sound evaluation is critical for positioning ships in the right regions for a given period or for planning fleet management (such as investing in new ships) for a longer term. This task is typically done in practice by comparing the overall growth in tonnage (or the number of ships) in the market with growth in global petroleum product trade in volume (barrels) (Clarkson, 2014). This paper provides an alternative method by modeling the market as a queueing system. This approach is effective especially when evaluating the impact of a change in ton-mile or a demand change in a specific route, instead of changes in the overall market.

Since significant gaps in profitability between regions are endemic in tanker shipping, controllers continually review the optimal allocation (or distribution) of their ships. We formulate this allocation problem as a utilization ratio game and compare the behaviors of controllers with different fleet sizes.

The controller manages several ships in a region and efficiently assigns available ships to irregularly arriving cargoes with an objective of maximizing profit over time. Based on the assignments decisions (or controls), the ships repeat laden voyages, ballast voyages and idling time over specific time periods. Ballast voyage and idling time are the largest costs for a controller, usually much greater than operating costs such as fuel and crew. In practice, such assignments are mostly done manually by a team of chartering managers (Lim, 2015). We formulate this assignment problem as a Semi-Markov problem, obtaining an optimal stationary policy, which maximizes average profit by rejecting some cargoes based on the location and status of ships.

Table 1.1: Clarkson's data

Column name	Description	Note
Date	Date of contract or report	
Name	Ship's name	
Built	Ship's built year	
Dwt	Ship's deadweight tonnage	40,000-50,000 tons
Hull Type	Double hull, double sides, etc.	Mostly double hull
Quantity	Cargo quantity	30,000-40,000 tons
Type	Cargo type	UMS, diesel, Gasoil, etc.
Charterer	Shipper or Shipping company	Shell, Total, BP, etc.
Laycan From	'pick up from' date	
Laycan To	'pick up to' date	
Load	Loading port	
Discharge	Discharging port	Discharging means 'unloading'
Rate	Freight rate (or price)	
Owner	Shipowner's name	

1.4 Data

We provide preliminary analysis of market data in order to illustrate the product tanker market. The dataset, provided by Clarksons, contains 17,097 spot transactions in the handysize product tanker market during January 1, 2011 - October 31, 2014. Handysize tanker refers to ships with the deadweight of 40,000 to 60,000 tons and with the actual cargo capacity of 30,000 to 40,000 tons. The data columns are shown in Table 1.1.

We don't know how completely the dataset represents the entire spot market transactions during the same period. A significant portion of the transactions are not reported because some transactions are done privately between shippers and controllers, or shipping companies choose not to disclose transaction details as they are conventionally reluctant to reveal their ship positions and status (Lim, 2015). The data shows that a total of 1,542 ships are owned by 344 different owners (each owner owns fewer than 5 ships on average).

Figure 1-1 is a histogram of the number of ships based on the average number of loads carried per year. A ship typically can carry at least one load in a month or twelve cargoes in a year (Lim, 2015). The data shows that only 14% of the ships

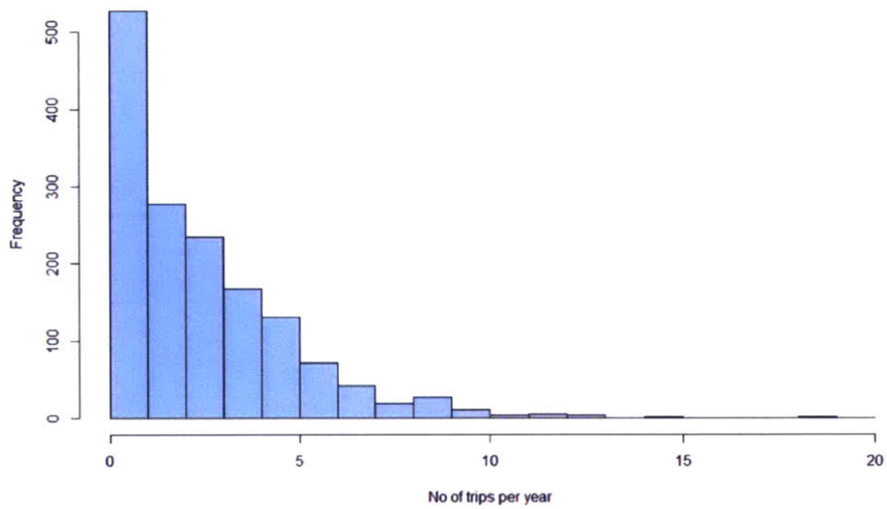


Figure 1-1: Histogram by number of loads carried

Table 1.2: Transactions by cargo type

Type	No. of transactions	% of total
UMS (unleaded motor gasoline)	5,272	31%
Gasoil	2,756	16%
Naphtha	2,366	14%
Jet	1,072	6%
Ultra Low Sulfur Diesel	641	4%
Diesel Oil	155	1%
Condensate	133	1%
Others	92	1%
Type not reported	4,610	27%
Total	17,097	100%

carried five or more loads per year on average. We cannot conclude this due to the incompleteness of the dataset but such figure implies overall low utilization of ships in the market.

Table 1.2 shows cargo types in the data ordered by the number of transactions.

Table 1.3 shows the top ten routes ranked by the number of transactions, which comprise 18% of the entire transactions. The route from the US Gulf region to the UK/Continent region (the Northern European continent plus the UK) had the highest volume, followed by the route from the Black Sea region to the Mediterranean market. In the data, there are a total of 2,123 cargo routes for fewer than 400 ports, implying

Table 1.3: Top ten routes

Loading	Discharging	No. of Transactions
USG	UKC	561
BLACK SEA	MED	387
SICILY	MED	385
UKC	USAC	318
WC INDIA	JAPAN	287
UKC	USG	238
TUAPSE	MED	230
ARA	USG	229
SARROCH	MED	227
S KOREA	SINGAPORE	221

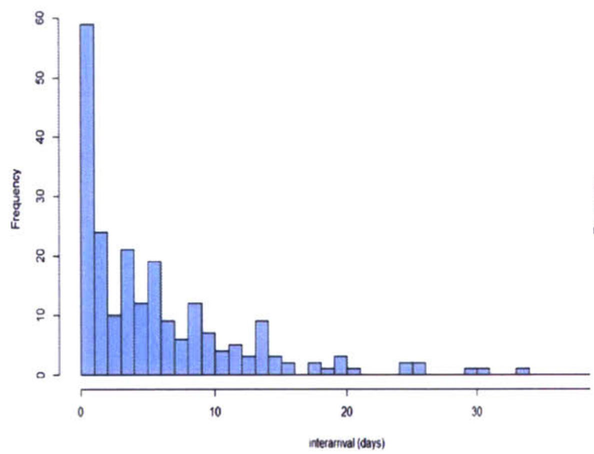


Figure 1-2: S Korea to Singapore

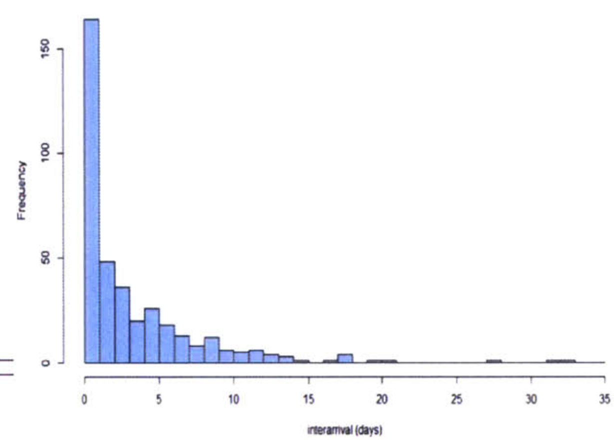


Figure 1-3: WC India to Japan

that the product tanker market forms a very sparse graph. Such sparsity indicates potential long empty voyages.

Figures 1-2 and 1-3 show the histograms of cargo interarrival times for the two busiest routes in the Asian market. As the charts show, interarrival times look to be exponentially distributed. The other major routes also produced a similar pattern. Based on this observation, we assume that cargo arrivals in the product tanker market follow Poisson processes. Such assumption provides a basis for our view of analyzing the market as a queueing system.

Chapter 2

Literature Review

2.1 Maritime transportation

Christiansen et al. (2004, 2013) reviewed current and past research related to maritime transportation and noted that it mostly deals with optimization problems in a static setting (static as opposed to dynamic or stochastic). Accordingly, mathematical programming (linear, integer or mixed) techniques are the main methodologies. This tendency can be attributed to the characteristics of the specific markets to which those research methods have been applied. In the survey of research papers of the past ten years, Christiansen et al. (2013) divided past research by industry, into liner shipping and tramp shipping.

In liner shipping, carriers, such as container shipping companies, run a regular service between fixed ports and on a published, fixed time schedule, which seldom changes. Hence, problems related to long-term planning (months or up to a year) are the dominant subject in this area. Specifically, network design and fleet deployment problems are the frequent subjects.

In tramp shipping, carriers are plying between different ports, depending on where they find suitable cargoes. The product tanker market, the subject of this thesis, falls under this category. Past research on this market has dealt with fleet size and composition problems. These are the problems of shipping companies that manage ships of various types and sizes and optimize their fleets over the long term by buying and selling ships or by chartering in and chartering out ships. Another frequent topic of research on tramp shipping is routing and scheduling problems. These types of problems arise when ships have to pick up and deliver cargoes within specific time windows. In this context, the tramp carrier typically has a set of mandatory cargoes, and will try to increase its revenue by transporting optional spot cargoes. The mandatory cargoes are based on agreements between the shipping company and the cargo owners. The key assumption (constraint) in this problem is that ships must carry all those mandatory cargoes, which distinguishes them from our research. We deal with the problems in the spot market, where product tanker companies carry spot cargoes with pickup deadlines based on short-term contracts[1]. If cargoes are not picked up by the deadlines, the contracts are cancelled without penalty.

The focus of the current research on the liner and contract-based tramp shipping markets is justified, given the size of those markets. According to Gorton et al. (2004), 70% of global goods are carried by non-liner shipping, of which only 5-10% is carried in the spot market. In this respect, product tanker shipping is a very specialized market, of which spot transactions constitute a large portion.

To summarize, unlike the mainstream of maritime research, this thesis studies the dynamic ship assignment problem with the assumptions that shipping companies have identical ships and have no long-term contracts. To the best of our knowledge, there are no published papers about the dynamic ship assignment problem in the maritime transportation industry.

2.2 Dynamic programming

Product tanker shipping resembles the truckload transportation market in the sense that loads of the same size (shiploads for tankers and truckloads for trucks) arrive with geographical and temporal uncertainty. Like tankers, truckload carriers transport a single shipper's cargo each time. Since demand is uncertain, a dynamic vehicle assignment problem arises — a truck dispatcher manages a fleet of trucks and assigns

[1]Contracts for spot cargo in tanker shipping are called "voyage charter contracts."

trucks to arriving loads with uncertainty.

The dynamic (or real-time) vehicle assignment problem in the truckload industry has been investigated by Powell (1987, 1986) and Godfrey and Powell (2002). In addition, it has been reported that working vehicle assignment solutions have been developed and implemented in the real market, improving profitability for large truck dispatchers according to Powell et al. (1988) and Simão et al. (2010).

In formulating the dynamic fleet management[2] problem, Godfrey and Powell (2002) approximated cost (or cost-to-go) functions with nonlinear (piecewise linear) functions. With such an approximation structure, cost functions are obtained by computer simulation. Such simulation-based dynamic programming is called approximate dynamic programming. In this thesis, we approximate the problem itself[3], instead of cost functions. We approximate our dynamic ship assignment problem with a queueing system and formulate the problem as a semi-Markov average cost problem. Hence, our solution is analytical (as opposed to simulation-based), in the sense that it is obtained by solving equations. The formulation and proof for such semi-Markov problems are provided in Bertsekas (2005).

2.3 Queueing model

Larson and Odoni (2007) introduce the formulation and the application of a spatial queueing model. A multiserver spatial queueing system, in particular, is called a "hypercube model." Urban service systems such as police cars and hospital ambulances qualify to be queueing systems because both customer arrivals (e.g., 911 calls) and service times (time from call to response) are random. With the assumption that both customer arrivals and service times are exponentially distributed, the hypercube model has been applied to police-sector design, response-area design for ambulances, demand-responsive delivery (pizza), and so on. Since the objective was to compute the workload of each vehicle in the assigned region, vehicles in Larson and Odoni

[2]Not to be confused with the long-term fleet management problem in Christiansen et al. (2013).
[3]Often called "problem approximation."

(2007) have identifications and are not anonymous. The anonymity of ships in this thesis is the key difference, making our formulation much simpler.

2.4 Game theory

Solving Nash equilibria in multiplayer games is an area of computer science. Daskalakis and Papadimitriou (2015) studied anonymous games from an algorithmic viewpoint. A general n-player game, in which each player has $\xi \geq 2$ strategies, is described in $n\xi^n$ numbers, an astronomical number for a large n. For such cases, a Nash equilibrium is not solvable. Anonymous multiplayer games, however, are polynomial in n for fixed ξ. Daskalakis and Papadimitriou (2015) introduce formulations and algorithms for such games and show that approximate mixed Nash equilibria in anonymous games can be computed in polynomial time. Their paper suggested to the present author that the formulation for our multi-ship or multi-controller game should take the form of dynamic programming. Solutions for both Bellman equations in dynamic programming and Nash equilibrium are the fixed points in a given space. We have solved Nash equilibria by what we call "strategy iteration," borrowing the name from "policy iteration" in dynamic programming.

2.5 General references

This thesis is an application of introductory theories taught in the following four classes at MIT (with textbooks cited in parenthesis): 1) Dynamic Programming and Stochastic Control (Bertsekas, 2005), 2) Applied Probability (Bertsekas and Tsitsiklis, 2002), 3) Logistical and Transportation Planning Methods (Larson and Odoni, 2007) and 4) Games, Decision and Computation. Textbooks and classnotes have been used as general references.

Clarkson (2014) publishes monthly business journals on the tanker shipping market, which we used as a basic introduction to the market. We also interviewed Lim (2015), who has over 20 years of experience in tanker and bulk ship chartering, for industry practice and for in-depth introduction to the market.

Chapter 3

Utilization Ratio Analysis

We evaluate the utilization ratio of the ships in a region in the product tanker market by approximating the region as a queueing system. This approximation is primarily based on the assumption that cargo demand at each port in the region follows independent Poisson processes. We interpret utilization ratio in two ways, first as an indicator of supply and demand of the market and second as an optimization measure for the controller. We validate the formulation by simulations and discuss applications and limitations of our queuing model. Finally, we formulate the utilization ratio game and compare the behaviors of controllers with different fleet sizes.

3.1 Definition

A utilization ratio (denoted by ρ) indicates how busy the ships are during a given period. For example, ρ of 0.7 indicates that a ship sails (loaded or empty) for 70 out of 100 days while idling for 30 days. By viewing cargo in the region as the customers (or users) and the ships as the servers, our queueing analysis models the region as an $M/M/m$ queue. M stands for 'memoryless' or exponential distribution for customer interarrival times or for service times. For our problem, *a service means pickup and delivery of a load.* The number of the ships in the region is denoted by m. Utilization ratio is defined

$$\text{utilization ratio}\,(\rho) = \frac{\text{total rate of cargo arrivals in the region}}{\text{total rate of service}}.$$

In this paper, utilization ratio is used in two different contexts with different assumptions about the pick-up deadline. From an analyst's viewpoint, the ratio indicates supply and demand of the market. From a controller's viewpoint, it predicts how the controller's ships are utilized.

3.2 Example

Let us illustrate the formulation using a simple example, in which two identical ships are serving in a region with four ports as shown in Figure 3-1. In this example, we attempt to evaluate supply and demand of the market by calculating the utilization ratio for the region. The solid lines represent laden voyages and the dashed lines represent ballast voyages. Cargoes move over two routes, from port 1 to 2 [route $(1,2)$] and from port 3 to 4 [route $(3,4)$]. For route $(1,2)$, 0.24 shiploads arrive per day on average and for route $(3,4)$, 0.16 shiploads arrive per day. As an example of a service, if a ship idles at port 4 when it is assigned to a load for route $(1,2)$, the ship must sail empty (ballast voyage) from port 4 to 1 and then sail loaded (laden voyage) from port 1 to 2. Hence, ballast voyage plus laden voyage constitutes one service and it takes 5 days. The ships are assigned to cargoes randomly, i.e., either available ship can be assigned to a load. Under such random assignment, it makes no difference whether the two ships are managed by a single controller or managed by two competitive controllers.

3.3 Assumptions

In order to approximate this region as a queueing system, we need the following assumptions: Cargo arrives according to an independent Poisson process. Sailing time between any two ports is deterministic. Loading and unloading are done in no

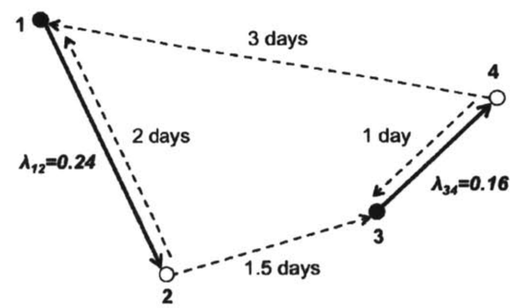


Figure 3-1: Example with four ports and two cargo routes

time and take zero days. Ships idle only at the discharging ports (port 2 and 4 in the example) and sail only when they are assigned to a load. Ships are anonymous and any ship available can be assigned to any load. Cargo has no pickup deadlines and unassigned loads enter a queue and then are served in a first-in, first-out (FIFO) manner. Service time for a load consists of ballast voyage and laden voyage and is a random variable because ballast voyage is uncertain. We assume it is exponentially distributed.

It is notable that there are two strong assumptions. The first one is related to service time distribution. Given the assumption that sailing time between any two ports is deterministic, service time is distributed by some probability mass function. However, for a queueing system approximation, we assume that service times of the ships are independently and identically (exponentially) distributed. The second assumption is related to the pickup deadline. In the real market, cargoes should typically be picked up by a certain deadline (cancellation date). However, since we intend to evaluate supply and demand of the region, we assume there is no deadline in this example. In other words, there is an infinite queue and cargoes are not lost even if there are no available ships.

3.4 Formulation

In order to formulate the queueing problem we introduce the following notation. The indices i, j, s, and t refer to specific ports.

$\mathcal{R}$	set of cargo routes (i, j) in the region
$\mathcal{D}$	set of discharging ports in the region
$\mathcal{L}(j)$	set of loading ports for shiploads discharged at port j
$\lambda_{i,j}$	Poisson arrival rate for shiploads from port i to j
λ	total cargo arrival rate in the region
μ	rate of service of a ship, i.e., picking up and transporting a shipload
ρ	utilization ratio
S	sailing (or service) time of a ship
$\tau_{i,j}$	(deterministic) sailing time from port i to j
π_j	(steady-state) probability of a ship idling at port j
$p_{i,j}$	(steady-state) probability of a ship serving a shipload from port i to port j

Based on the assumption of independent cargo arrivals, total demand (denoted by λ), or total rate of cargo arrivals in the region, is the sum of all arrival rates, i.e., $\lambda = \sum_{(i,j)\in\mathcal{R}} \lambda_{i,j}$. The total rate for our example is 0.4.

If the ships are randomly assigned to loads, which arrive according to independent Poisson processes, the probability that a ship is idling at port j while available for assignment in the long run is

$$\pi_j = \frac{\sum_{i\in\mathcal{L}(j)} \lambda_{i,j}}{\lambda} \quad \text{for all } j \text{ in } \mathcal{D}.$$

This is due to the memoryless property of exponential distribution and can be shown by calculating steady-state Markov chain probability.

Hence, the mean service time for serving a shipload from port s to port t is

$$\tau_{s,t} + \sum_{j\in\mathcal{D}} \pi_j \tau_{j,s} \quad \text{for all } (s,t) \text{ in } \mathcal{R} \tag{3.4.1}$$

where $\tau_{s,t}$ and $\tau_{j,s}$ are the deterministic sailing times between ports s and t and between ports j and s, respectively. The second term in (3.4.1) represents the mean ballast voyage time to serve a load from port s to t.

Due to the memoryless property of exponential distribution, the probability that a ship is serving a load from port s to t in a steady state is

$$p_{s,t} = \frac{\lambda_{s,t}}{\lambda} \quad \text{for all } (s,t) \text{ in } \mathcal{R}.$$

Hence, the mean service time becomes

$$\begin{aligned} \mathbb{E}[S] &= \sum_{(s,t)\in\mathcal{R}} p_{s,t}(\tau_{s,t} + \sum_{j\in\mathcal{D}} \pi_j \tau_{j,s}) \\ &= \sum_{(s,t)\in\mathcal{R}} p_{s,t}\tau_{s,t} + \sum_{(s,t)\in\mathcal{R}} \sum_{j\in\mathcal{D}} p_{s,t}\pi_j\tau_{j,s} \qquad (3.4.2) \\ &= \text{laden voyage} + \text{ballast voyage}. \end{aligned}$$

As (3.4.2) shows, mean service time is the sum of the mean laden voyage time and the mean ballast voyage time.

As previously stated in Section 3.3, we purposely assume that service times are independently and exponentially distributed. Hence, the rate for the exponential distribution (denoted by μ) is the reciprocal of this mean service time, i.e., $\mu = 1/\mathbb{E}[S]$.

Finally, the utilization ratio is

$$\rho = \frac{\lambda}{n\mu} = \frac{\lambda\mathbb{E}[S]}{n} \qquad (3.4.3)$$

where n is the number of the ships in the region. Since the mean service time $\mathbb{E}[S]$ is the sum of laden voyage and ballast voyage, we can rewrite (3.4.3) as

$$\rho = \rho_L + \rho_B$$

where ρ_L is the laden voyage ratio and ρ_B is the ballast voyage ratio, and by the definition of the utilization ratio, we have

$$\rho_I + \rho_L + \rho_B = 1$$

where ρ_I is the idle time ratio. Laden voyage, ballast voyage and idle time ratios indicate how much time a ship spends in sailing loaded, sailing empty and idling,

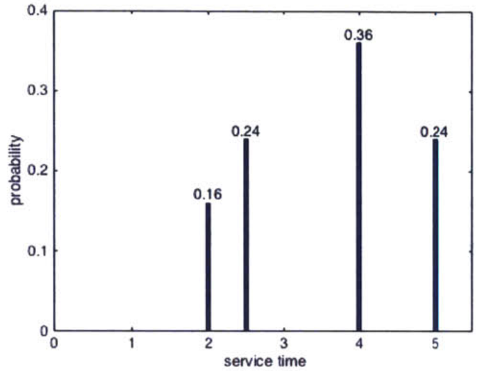


Figure 3-2: Pmf of service times

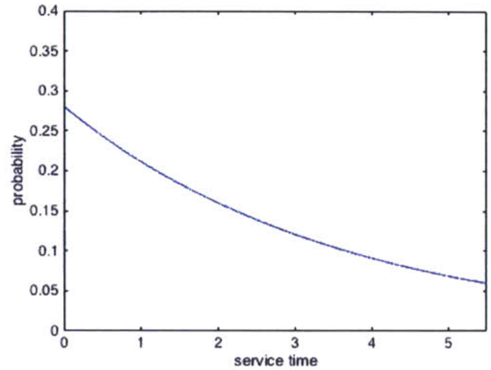


Figure 3-3: Exponential distribution

respectively, during a given period.

3.5 Validation

Applying (3.4.2) to our example in Figure 3-1, we obtain the mean service time by

$$\mathbb{E}[S] = \frac{\lambda_{1,2}}{\lambda}\tau_{1,2} + \frac{\lambda_{3,4}}{\lambda}\tau_{3,4} + \frac{\lambda_{1,2}^2}{\lambda^2}\tau_{2,1} + \frac{\lambda_{1,2}\lambda_{3,4}}{\lambda^2}(\tau_{2,3} + \tau_{4,1}) + \frac{\lambda_{3,4}^2}{\lambda^2}\tau_{4,3}. \tag{3.5.1}$$

With $\lambda = \lambda_{1,2} + \lambda_{3,4}$, we can rewrite the mean service time in (3.5.1) as

$$(\tau_{1,2} + \tau_{2,1})\frac{\lambda_{1,2}^2}{\lambda^2} + (\tau_{1,2} + \tau_{4,1})\frac{\lambda_{1,2}\lambda_{3,4}}{\lambda^2} + (\tau_{3,4} + \tau_{2,3})\frac{\lambda_{1,2}\lambda_{3,4}}{\lambda^2} + (\tau_{3,4} + \tau_{4,3})\frac{\lambda_{3,4}^2}{\lambda^2}. \tag{3.5.2}$$

The first two terms in (3.5.2) represent the mean service time of serving shiploads from port 1 to 2. The remaining two terms represent the mean service time of serving shiploads from port 3 to 4. When a load arrives at the region while there is an available ship (or ships), the ship is *randomly* located either at port 2 or at port 4. The fractional terms in (3.5.2) correspond to the probability of each event in the probabilistic sample space. For example, the fraction in the first term, $\lambda_{1,2}^2/\lambda^2$, represents the probability of an event that a ship is located at port 2 when it is assigned to a load from port 1 to 2, in which case, the service time is 4 days ($= \tau_{1,2}+\tau_{2,1}$). The probability mass function of service times (S) for the example is shown in Figure 3-2.

Given the probability distribution, the mean service time for the example equals

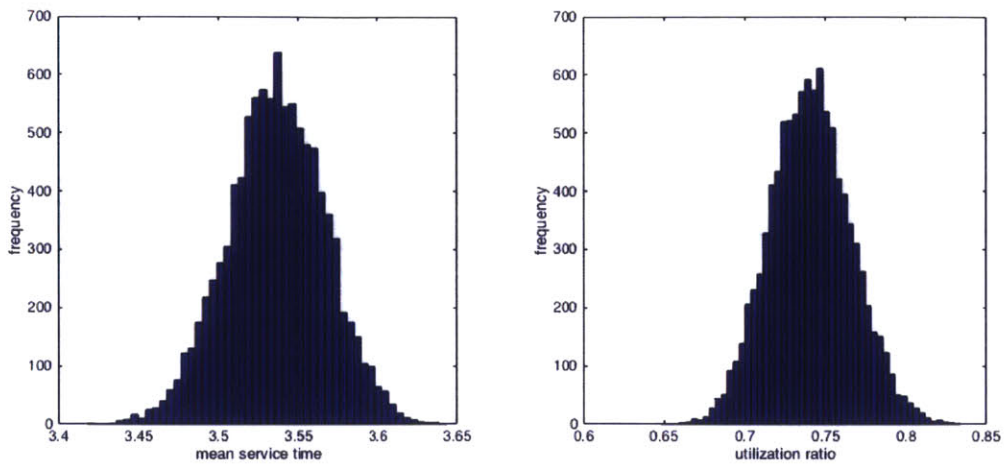


Figure 3-4: Histogram of simulation result

3.56, i.e., it takes 3.56 days on average for the ships to transport a load in this region. The utilization ratio computes to 0.712, i.e., the ships are busy (sailing) during 71% of the time.

We ran a simulation for 10,000 times to test the formulation and plot a histogram of the result as shown in Figure 3-4. Each simulation generated 1,000 cargoes according to the arrival rates and assigned the ships randomly. The mean service time in the simulation is 3.54 days and the utilization ratio is 0.741. While our estimate for the mean service time is close to the simulation result, the histogram shows that our model has a high probability to underestimate the utilization ratio. In other words, the ships in the simulation were busier than expected. Such gap between estimation and simulation result is obviously because we assume that service times follow exponential distribution, which is different from the probability mass function shown in Figure 3-2. In the pmf, the probability of service time being longer than the average is higher than the probability of being shorter than the average. Our exponential distribution would assume the opposite (see Figure 3-3).

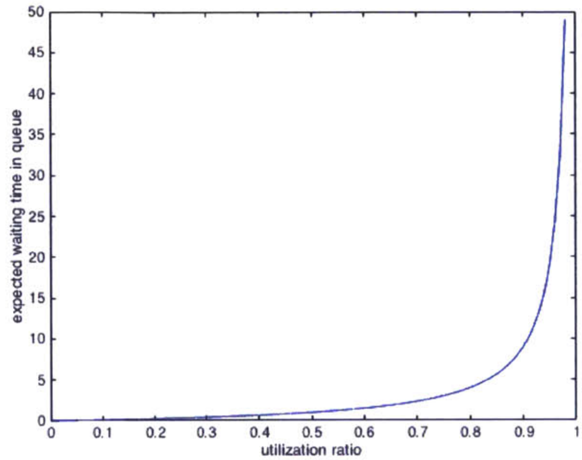


Figure 3-5: Waiting time in queue

3.6 Application to the real market

3.6.1 Sensitivity

We claim that utilization ratio analysis can be used for evaluating supply and demand of the market. Can a specific utilization ratio indicate whether the market is oversupplied or not? The answer can be obtained when the model is applied and tested using the real market data, which is beyond the scope of this paper, but queueing theory helps us to understand that *the tanker market becomes highly sensitive to changes in demand when levels of utilization are high.* Figure 3-5[1] illustrates this phenomenon that the expected waiting time in queue for a user (cargo) grows exponentially as the utilization ratio becomes close to 1. Waiting time represents the delay between the time a cargo arrives at the market and when it is actually picked up. Hence, a very long waiting time means the market suffers a severe shortage of ships. In such a case, our queueing model shows that the market (price) will fluctuate even with a small change in demand.

[1] For an M/M/1 queue with $\mu = 1$.

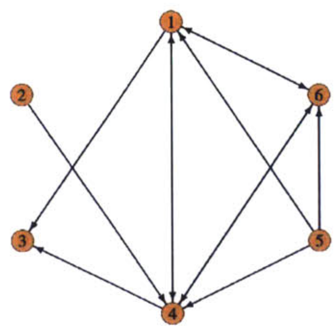


Figure 3-6: Laden voyage routes

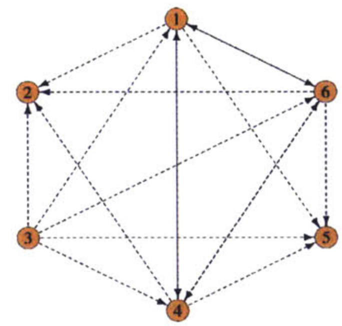


Figure 3-7: Ballast voyage routes

3.6.2 Ton-mile

The tanker market participants view ton-mile, one ton of cargo carried one nautical mile, as an important driver of demand. Increase in ton-mile is typically understood as growth in cargo volume for long-haul routes or emergence of new cargo routes that increase overall sailing time and distance. From the queuing model point of view, a change in ton-mile can be interpreted as a change in service time (or utilization ratio), thus our model can directly estimate the impact of change in ton-mile.

To illustrate this idea, we introduce another example as shown in Figure 3-6 and in Figure 3-7. There are six ports and twelve cargo routes in the region. The sailing time between ports is two, three, or four days. For example, it takes two days for route $(1, 2)$, three days for route $(1, 3)$, and four days for route $(1, 4)$. There are three identical ships in the region. Cargo arrivals for each cargo route follow the identical distribution (Poisson) with the rate (denoted by p) of 0.04 shiploads per day. Hence, the total arrival rate (λ) is 0.48, i.e., a new shipload arrives at the market every 2.08 days. Based on the same assumption stated in Section 3.3, the utilization ratio computes to 0.898. Hence, as shown in Figure 3-5, the market is highly sensitive to even a small change in the utilization ratio. The laden voyage ratio equals 0.44 and the ballast voyage ratio equals 0.458.

Now suppose the overall demand (λ) in the market increases by 1.0%. But the rates of change are not uniform over different routes. The demand for routes $(1, 4)$ and $(4, 1)$, the long-haul cargoes, increases by 18.5% while the demand for routes $(1, 6)$, $(3, 4)$, $(5, 4)$, $(5, 6)$ and $(6, 1)$, the short-haul cargoes, decreases by 5.0%, while demand for the remaining routes remain unchanged. Can we expect that the utilization ratio to increase by 1.0% in line with the overall demand growth?

Our queueing model says otherwise. The new utilization ratio equals 0.918, a 2.3% increase and the laden voyage ratio increases by a greater 3.0%. It is because the growth in long-haul demand not only increased the overall demand but also raised the mean service time. Given the already very high utilization ratio base, we can expect the change in demand will cause a much worse supply shortage.

3.7 Deadline

In our two previous examples, we assumed an infinite queue capacity. This means that shiploads are not lost even if no ships can pick up the shiploads within deadlines, and instead, they are postponed. Such assumption makes sense when we evaluate supply and demand of the market using the utilization ratio. However, we need to revise the assumption when a controller attempts to evaluate its own utilization ratio in a competitive region. Since there are competitors, cargoes are highly likely to be lost if not picked up within deadlines. Unfortunately, it is impossible to accurately model such situation using queueing theory. One alternative is to model the problem as an $M/M/m/K$ queue. The capacity of the queueing system is denoted by K.

We use the same region with six ports as shown in Figure 3-6 and Figure 3-7 as an example to illustrate this modeling issue. The controller manages three identical ships in the region and wants to evaluate the utilization ratio of its ships. We assume that all cargoes have a deadline of 4 days. If no ships are available within the deadline, cargoes are lost. We attempt to model this problem as an $M/M/3/3$ queue. This means the queuing capacity is zero and cargoes are lost immediately when no ships are available. All the other assumptions stated in 3.3 remain valid for this problem.

Since cargoes are lost when all of the three ships are busy, we need to calculate the probability of such event (denoted by P_m) by Erlang's loss formula (Larson and Odoni, 2007)

$$P_m = \frac{(\lambda/\mu)^m/n!}{\sum_{i=0}^{m}(\lambda/\mu)^i/i!}.$$

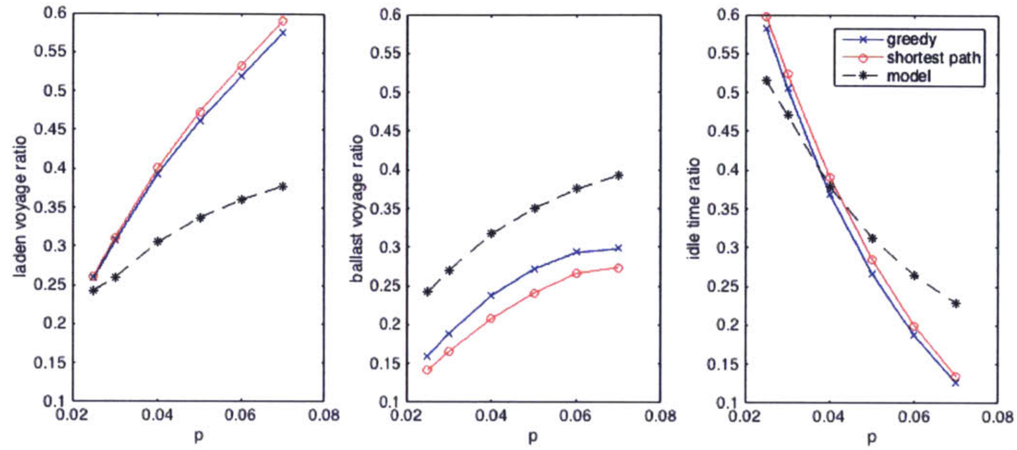


Figure 3-8: Utilization ratio in simulation

Total demand in the region is now $\lambda(1 - P_m)$. Therefore, the utilization ratio is

$$\rho = \frac{\lambda(1 - P_m)}{n\mu}.$$

We test the formulation by simulation. In this simulation, time is discretized and it advances in one day intervals. Shiploads for each route are generated at each time interval according to Bernoulli process (as a discrete version of Poisson) with a success rate of p and are cancelled if they are not picked up by an available ship by the deadline. The duration of each simulation is 365 days and we repeat it for 100 times. Simulations are done for different values of arrival rate p. We use two assignment methods for each cargo in the simulation, 'greedy assignment' and 'shortest path algorithm.' The greedy method immediately assigns the closest available ship to a cargo, which is similar to the random assignment assumption that we use to calculate the utilization ratio. We will explain these methods in detail in Section 4.3.3.

The result in Figure 3-8 shows that our model significantly underestimates the laden voyage ratio while overestimating the ballast voyage ratio. The reason for the error is that our model excludes the possibility of not immediately assigned shiploads being picked up by the ships that become available within the four-day deadline. This explains why the error for the laden voyage ratio is greater for high values of p.

3.8 Utilization ratio game

So far we have evaluated utilization ratio for a single region. Now imagine a market divided into several regions where multiple controllers are competing. Ships can move between regions (freely or for a cost) and controllers distribute their ships over different regions with an objective of maximizing utilization ratios. We view this game from two different perspectives, depending on how we define a player. In the first perspective, the players are individual ships that choose a region to maximize the utilization ratio. In the second perspective, the players are individual controllers who have one or more ships and maximize the average utilization ratio.

For analysis of this anonymous, multiplayer game, we assume a complete information setting[2].

3.8.1 Individual ships as players

In this market, there are n ships and ξ regions and ships can move *freely* between regions. We define demand-service ratio, denoted by $\hat{\rho}^l$, a ratio of total demand rate (λ_l) and total service rates (μ_l) for region l. It corresponds to utilization ratio if there is only one ship in the region. The higher the ratio is, the more ships are demanded. We also introduce the following notation:

$\hat{\rho}^l$	demand-service ratio for region l
$[n]$	set of players, $[n] = \{1, 2, ..., n\}$
$[\xi]$	set of pure strategies (regions), $[\xi] = \{1, 2, ..., \xi\}$
Δ^ξ	set of distribution over $[\xi]$
δ	mixed strategy profile
δ_i	mixed strategy of player i in δ
δ_{-i}	collection of mixed strategies except for i's in δ
$\delta_i(l)$	probability of player i being positioned in region $l \in [\xi]$

Pure strategy (action) of a player (ship) in this game corresponds to the region

[2]In a complete information game, the number of players, a finite set of pure strategies for each player, and a utility function of each player are known to all players.

in which the ship is positioned. Each player uses a randomized (or mixed) strategy (denoted by δ_i), i.e., a strategy takes the form of a distribution over $[\xi]$. For example, $\delta_i(1)$ means the probability of player i being positioned in region 1. Mixed strategy profile (denoted by δ) is the collection of strategies of all players, i.e., it is a mapping of $[n]$ onto Δ^ξ. Since any ship can go to any region in the market, all players have the same strategy set.

Utility of player i when positioned in region l is calculated as

$$u_l^i = \frac{\hat{\rho}_l}{n_l + 1}, \text{ for all } i \in [n] \tag{3.8.1}$$

where n_l is the number of players except for player i choosing strategy l. Since all players have the same strategy set $[\xi]$ and as in (3.8.1) utility can be written as a function of the number of players other than i choosing strategy $l \in [\xi]$, this game is symmetric and there exists a symmetric Nash equilibrium (Nash, 1951). Hence, we have $\delta_1 = \delta_2 = \ldots = \delta_n$, i.e., all players have the same mixed strategy (denoted by δ^*) and n_l becomes a binomial random variable.

Finally, from the fact that all players share the same mixed strategy, the Nash equilibrium simply is

$$\delta_i^*(l) = \frac{\hat{\rho}^l}{\sum_{k=1}^{\xi} \hat{\rho}^k}, \text{ for all } i \in [n].$$

The Nash equilibrium suggests that positioning a ship following the distribution of demand (or $\hat{\rho}$) is the best response of any player. This obvious result explains 'herd mentality'; a rational player should do exactly what other players do — positioning more ships in the market with more demand — and unilaterally deviating from this strategy will only deteriorate its payoff.

3.8.2 Controllers as players

In this section, controllers who have multiple ships are the players. Since different players have different pure strategy sets depending on the number of ships they control, the game is no longer symmetric. We discuss such games using a simple example where 100 'small' players who have only one ship each compete with a single 'big' player who has 100 ships. The market consists of two regions, namely region 1 and region 2. The demand-service ratios for each market, $\hat{\rho}^1$ and $\hat{\rho}^2$, are given as parameters. For this example, we assume that *there is a fixed cost for switching regions.*

We introduce the following notation:

$\mathcal{B}$	set of pure strategies of big player, i.e., $b_j \in \mathcal{B}$
b_j	pure strategy of big player, $j = 0, 1, 2, ..., 100$
b_j^l	number of big player's ships in region $l \in \{1, 2\}$ for pure strategy b_j
$[\xi]$	set of pure strategies of small player
δ_i	mixed strategy of small player $i \in \{0, 1, 2, ..., 100\}$
δ_i^l	probability of small player i being in region l

A specific pure strategy of the big player is denoted by $b_j = (b_j^1, b_j^2)$ where b_j^1 and b_j^2 represent the number of ships in region 1 and region 2, respectively. The big player has 101 pure strategies, i.e., $\mathcal{B} = \{(0, 100), (1, 99), (2, 98), ..., (100, 0)\}$. Small players have two pure strategies, i.e., $[\xi] = \{1, 2\}$. The specific mixed strategy of small player i is denoted by $\delta_i = (\delta_i^1, \delta_i^2)$ where δ_i^1 and δ_i^2 represent the probabilities of its ship being in region 1 and region 2, respectively. The big player plays one of the pure strategies and small players play mixed strategies in this game.

We evaluate the Nash equilibrium in the following situation:

- In the initial state, the ship demand in region 1 and region 2 are equal, i.e., $\hat{\rho}^1_{(0)} = \hat{\rho}^2_{(0)} = 90$. Accordingly, both the big player and small players evenly distribute their ships in those two regions, i.e., the big player plays $(50, 50)$ and small players play $(0.5, 0.5)$.

- In the new state, the demand in region 1 becomes twice as big as the demand in region 2, while overall demand remains unchanged. The demand of region 1 and region 2 are now $\hat{\rho}^1 = 120$ and $\hat{\rho}^2 = 60$, respectively.

We want to find the Nash equilibrium and utilization ratio for each player in the new state. We do this by iteratively searching over the pure strategies of the big player

Table 3.1: Nash equilibria

	Equilibrium (region 1)		Utilization ratio	
Switching cost	Big player	Small player	Big player	Small player
0.0	0.667	0.667	0.9	0.9
0.05	0.641	0.67	0.8925	0.8754
0.1	0.615	0.67	0.8861	0.8524
0.3	0.5	0.67	0.8745	0.7755

and over the mixed strategies of small players (we call this "strategy iteration"). We include the formulation for the strategy iteration in Appendix A.

We obtained Nash equilibria and the corresponding utilization ratios with varying switching cost. The unit of cost is utilization ratio. For example, a switching cost of 0.1 means that switching from one region to another will cost a ship 0.1 utilization ratio.

As Table 3.1 shows, if there is no switching cost, the equilibrium suggests that both the big player and small players will distribute their ships according to the demand-service ratio distribution (2/3 for region 1 and 1/3 for region 2). The big player will position two-thirds of its ships in region 1, and small players will position its ship to region 1 with a probability of 0.667. Hence, the utility of the big player and small players are the same.

As switching cost increases, big player's allocation to region 1 does not increase as much as the demand grows. Eventually, when switching cost is greater than 0.3, the big player stays in its initial distribution $(50, 50)$. On the other hand, small players will always distribute their ships according to the demand distribution. As a result, the utilization ratio of small players sharply decreases as the switching cost grows while the average utilization ratio of the big player remains high (see Figure 3-9).

The result reveals that a large controller who operates a number of ships will react to a change in demand more conservatively than smaller players. In other words, the large controller causes a shortage in ships by not supplying enough ships to the regions with high demand. Small controllers, in contrast, will distribute their ships according to the distribution of market demand. As a result, the large player obtains greater utilization ratio. It may be counter-intuitive to find that the Nash equilibrium in our

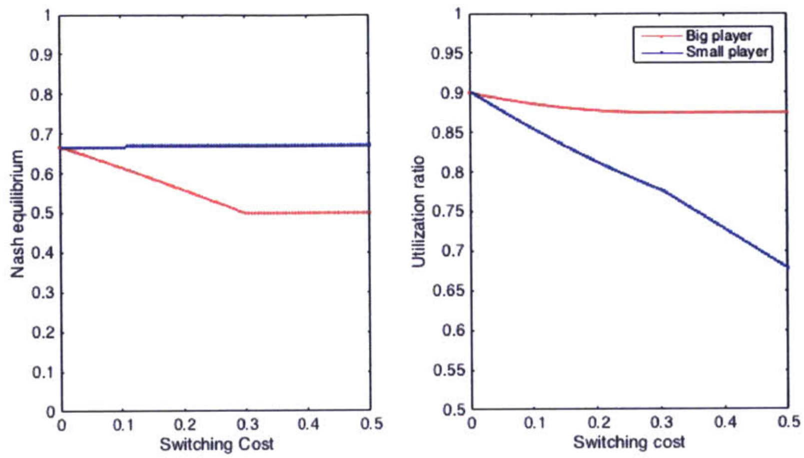


Figure 3-9: Nash equilibria

example suggests that big player will not aggressively move ships to region 1 (high market) since it has the lower average switching cost per ship compared to that of a small player. However, such strategy makes sense given the fact that it already has exposure (ships) in both regions and thus it does not have to make aggressive allocation changes, which will only incur unnecessary switching cost. In contrast, it is the only option for small players to switch to a better market despite the switching cost because the opportunity cost (cost of not moving) is greater than switching cost.

Chapter 4

Dynamic Ship Assignment

In this final problem, a controller assigns ships to irregularly arriving cargoes with the objective of maximizing profit over time. With the assumption that both cargo inter-arrival times and ship service times are exponentially distributed, the tanker market is viewed as a queueing system, and the problem is approximated as a Semi-Markov problem that minimizes average cost (or maximizes average profit) over an infinite horizon. The stationary policy obtained from Bellman equations for the problem suggests cargo rejections, depending on the status and the location of ships. Despite rejections, the stationary policy yields the best profit per load among alternative assignment methods when profit margins for each load are low. On the contrary, when the margins are high, myopic greedy assignment method yields comparable results. The stationary policy also suggests that the controller needs to be selective even when demand is low in order to maximize profit.

4.1 Example

We use the same example used in Section 3.6.2 — a hypothetical region with six ports — to illustrate this optimal control problem. We also use the same notation introduced in Section 3.4 to describe the region in the example and formulate the mean service time. In Figure 4-1 the solid lines represent laden voyage (cargo) routes and the dashed lines represent ballast voyage (empty voyage) routes

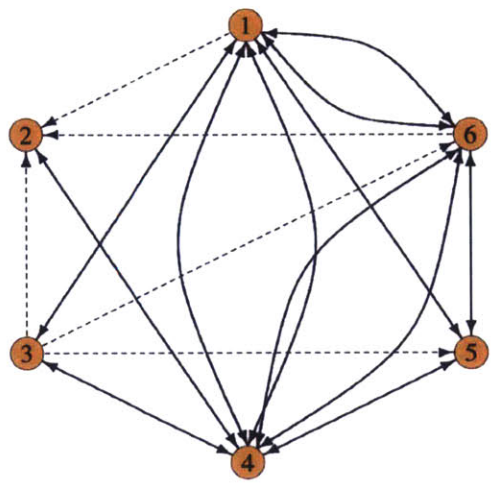


Figure 4-1: Region with six ports and twelve cargo routes

traversed to pick up cargoes. There is a total of twelve cargo routes, i.e., $\mathcal{R} = \{(1,3),(1,4),(1,6),(2,4),(4,1),(4,3),(4,6),(5,1),(5,4),(5,6),(6,1),(6,4)\}$. The controller has three identical ships.

This region forms a graph that has far fewer edges (or cargo routes) than the maximum number of edges (30) for a six-node graph. The real product tanker market also typically forms such sparse graphs and thus ships inevitably experience long ballast voyages. For example, port 3 in Figure 4-1 has only incoming cargoes without backhaul (or outgoing) cargoes, so ships that discharged cargo at port 3 must make a ballast voyage to pick up the next cargo.

4.1.1 Assumptions

Sailing time between any two ports is deterministic. Loading and unloading are done in no time and take zero days. Ships idle only at the discharging ports ($\mathcal{D} = \{1,3,4,6\}$) and sail only when they are assigned to a load. Shiploads sailing along the twelve cargo routes arrive according to independent Poisson processes. Shiploads have pickup deadlines; any shiploads unassigned by those deadlines are lost forever. Time to process a load consists of ballast voyage (cost to the controller) and laden voyage and is distributed by some probability mass function, as ballast voyage is a random variable. However, for a Semi-Markov formulation, it is assumed service times are *independently and exponentially* distributed.

While the remaining assumptions are the same as those stated in Section 3.3, there is one important distinction: we assume that service time for each route follows a different exponential distribution. In Section 3.3, we assume that all service times, regardless of the route, follow the same exponential distribution.

4.1.2 Mean service time

Cargoes arrive according to independent Poisson processes. If ships are randomly assigned to a load, the probability that a ship is located at port j while available for assignment in the long run is $\pi_j = \frac{\sum_{i \in \mathcal{L}(j)} \lambda_{i,j}}{\lambda}$ for all j in $\mathcal{D}$. Hence, the mean service time for serving the load from port s to t is

$$\mathbb{E}[T_{s,t}] = \tau_{s,t} + \sum_{j \in \mathcal{D}} \pi_j \tau_{j,s} \quad \text{for all } (s,t) \text{ in } \mathcal{R}$$

where $\tau_{s,t}$ and $\tau_{j,s}$ are the deterministic sailing times between ports s and t and between ports j and s, respectively. We denote the mean service $E[T_{s,t}]$ by $\overline{T}_{s,t}$. We assume the rate for exponential distribution of service time for route (s,t) to be the reciprocal of this mean service time, i.e., $\overline{T}_{s,t}^{-1}$.

It is worth pointing out that the mean service times for each cargo route need to be calibrated during simulations when we apply different assignment methods since a certain method will change the probability that a ship idles at port j (denoted by π_j). However, we present the simulation results in Section 4.3 without such calibration.

4.2 Formulation

We approximate our problem as a semi-Markov problem by assuming that both cargo interarrival times and service times are exponentially distributed. In semi-Markov problems, the time between successive controls (or state transitions) is a continuous random variable and cost is continuously accumulated. For example, in queueing systems state transitions correspond to arrivals or departures of customers. Because we essentially approximate the tanker market as a queueing system, for our ship assignment problem, *state transitions occur when a new load arrives at the region.*

We formulate our problem as a semi-Markov average cost problem and obtain a stationary policy by solving the corresponding Bellman equation. In dynamic programing, a policy (denoted by π by convention) is a sequence of functions (denoted by μ_k) that map states onto controls, i.e., functions that output a control (or decision) based on a given state. A *stationary* policy minimizes the average cost per stage over an infinite horizon and contains a function (denoted by μ) that outputs an optimal control based on a given state, *irrespective of the time of the control.* Hence, controls in the stationary policy for our problem assign the ships to a new load based on the given state, minimizing the average cost (or maximizing average profit) over an infinite horizon. A state includes the route for a new load and the status and location of the three ships, as explained in the following section.

4.2.1 State definitions

A state is defined as a quadruple of origin-destination pairs $(s,t)(s_1,i)(s_2,j)(s_3,k)$ where (s,t) is an arriving load required to be shipped from port s to t. The $(s_1,i)(s_2,j)$ (s_3,k) means the three ships are serving cargoes from port s_1 to i, from s_2 to j and from s_3 to k, respectively. A ship idling at port i is represented by $(0,i)$. For example, the state $(s,t)(0,i)(0,j)(m,n)$ means that two ships are idling at ports i and j and one ship is serving the load (m,n), when a new load (s,t) is required to be picked up. It is notable that the state does not contain the exact location of the ship serving the load (m,n). Because we assume exponentially distributed service times, the Bellman equations do not require such information (see Section 4.2.3). Hence, the state space (denoted by $\mathcal{S}$) become considerably reduced compared to the complexity of the actual problem.

The total number of states is $O(|E|^{N+1})$ where N is the number of the ships under control. We have total 5,424 states for the problem as ships are anonymous. The state space is finite and forms a single recurrent class.

4.2.2 Controls

For a given load, the controller decides on its assignment to one of the available ships. The controller may reject this cargo even if there is an available ship. For example, the controller has three controls for the state $(s,t)(0,i)(0,j)(m,n)$: 'assign the ship at i to the new load (s,t)', 'assign the ship at j' and 'reject'.

4.2.3 Bellman equation

In this problem we want to attain stationary policies that minimize the average cost per stage (or per day) over an infinite horizon (we call this "original problem"). We do this *by solving an equivalent stochastic shortest path problem* (Bertsekas, 2005). Consider a cycle starting from one of the states (by convention state $\boldsymbol{n}$) and returning to the same state $\boldsymbol{n}$ for the first time. The special state $\boldsymbol{n}$ is recurrent in the Markov chain corresponding to each stationary policy. Over an infinite horizon, such cycle repeats indefinitely. Each cycle can be viewed as a state trajectory of a corresponding stochastic shortest path problem with the termination state being $\boldsymbol{n}$. The average cost per stage equals the total cost for the cycle divided by the number of the stages. Finally, by minimizing this average cost per stage, we attain the stationary policy for the original problem.

We introduce the following notation for the formulation (Bellman equation). We use the boldface indices $\boldsymbol{i}$ and $\boldsymbol{j}$ to refer to specific states (boldface to avoid confusion with the indices for ports). We also use the underlined index $\underline{u}$ to refer to specific controls.

$h^*(\boldsymbol{i})$	optimal cost for the stochastic shortest path when starting at state $\boldsymbol{i}$
λ^*	optimal average cost per stage
$G(\boldsymbol{i},\underline{u})$	expected one-stage cost for state $\boldsymbol{i}$ and control $\underline{u}$
$\overline{\tau}_{\boldsymbol{i}}(\underline{u})$	expected transition time corresponding to $(\boldsymbol{i},\underline{u})$
$\underline{p}_{\boldsymbol{ij}}(\underline{u})$	transition probability from state $\boldsymbol{i}$ to $\boldsymbol{j}$ given the control $\underline{u}$
$U(\boldsymbol{i})$	set of controls that depend on the current state $\boldsymbol{i}$
μ	optimal function in the stationary policy, i.e., $\pi = \{\mu, \mu, \mu, ...\}$

For the equivalent stochastic shortest path problem (terminal state being $\boldsymbol{n}$), Bellman equation takes the form

$$h^*(\boldsymbol{i}) = \min_{\underline{u} \in U(\boldsymbol{i})} \left[G(\boldsymbol{i}, \underline{u}) - \lambda^* \overline{\tau}_{\boldsymbol{i}}(\underline{u}) + \sum_{\boldsymbol{j}=1}^{\boldsymbol{n}} \underline{p}_{\boldsymbol{ij}}(\underline{u}) h^*(\boldsymbol{j}) \right]. \tag{4.2.1}$$

Since the one stage cost in (4.2.1) is $G(\boldsymbol{i}, \underline{u}) - \lambda^* \overline{\tau}_{\boldsymbol{i}}(\underline{u})$, the cost function $h^*(\boldsymbol{i})$ has the interpretation of *differential cost.* It is the minimum of the difference between the expected cost to reach $\boldsymbol{n}$ from $\boldsymbol{i}$ for the first time and the cost that would be incurred if the cost per stage was the average λ^*.

Let us apply (4.2.1) to our example shown in Section 4.1. We introduce the following additional notation for this example. The indices i, j, m, n, s, t, q, r, u and v refer to specific ports.

b	charter hire (or gross profit) per day
c	operating (sailing) cost per day
r	total cargo arrival rate in the region
$\overline{T}_{s,t}$	mean service time for load (s, t)
$p_{u,v}$	(steady-state) probability of a new load (u, v) arriving at the region

If the controller assigns the new load (s, t) to a ship idling at port i (a control denoted by $\underline{u}'$), the ship must sail ballast from port i to s and then sail laden from port s to t. The cost of this assignment equals the charter hire deducted by the sailing cost, i.e., $G(\cdot, \underline{u}') = -b\tau_{s,t} + c(\tau_{i,s} + \tau_{s,t})$ where $\tau_{s,t}$ and $\tau_{i,s}$ are the deterministic sailing times between ports s and t and between ports i and s, respectively. We assume that such cost is incurred at the time of the assignment, not over continuous time from assignment to job completion. We also assume that there is no idling cost. The total cargo arrival rate in the region is denoted by r and is the sum of all cargo arrival rates, i.e., $r = \sum_{(i,j) \in \mathcal{R}} \lambda_{i,j}$. The probability of a new load (u, v) arriving at the region is $\lambda_{u,v}/r$.

For our example, we have three Bellman equations for three different events: 1) three ships are idling, 2) two ships are idling and 3) one ship is idling, when the new load (s, t) arrives.

For the event that three ships are idling when the load (s,t) arrives, i.e., for the state $(s,t)(0,i)(0,j)(0,k)$, the controller has the following four controls:

1. assign the ship at port i to the new load (s,t),
2. assign the ship at port j,
3. assign the ship at port k, and
4. reject the load.

The Bellman equation is

$$\begin{aligned}
h^*((s,t)(0,i)(0,j)(0,k)) = min\Big\{ & -b\tau_{s,t} + c(\tau_{i,s} + \tau_{s,t}) - \lambda^* \frac{1}{r} + \sum_{(u,v)\in\mathcal{R}} p_{u,v} \\
& \Big(\frac{r}{r + \overline{T}_{s,t}^{-1}} h^*((u,v)(0,j)(0,k)(s,t)) \\
& + \frac{\overline{T}_{s,t}^{-1}}{r + \overline{T}_{s,t}^{-1}} h^*((u,v)(0,j)(0,k)(0,t))\Big), \\
& -b\tau_{s,t} + c(\tau_{j,s} + \tau_{s,t}) - \lambda^* \frac{1}{r} + \sum_{(u,v)\in\mathcal{R}} p_{u,v} \\
& \Big(\frac{r}{r + \overline{T}_{s,t}^{-1}} h^*((u,v)(0,i)(0,k)(s,t)) \\
& + \frac{\overline{T}_{s,t}^{-1}}{r + \overline{T}_{s,t}^{-1}} h^*((u,v)(0,i)(0,k)(0,t))\Big), \\
& -b\tau_{s,t} + c(\tau_{k,s} + \tau_{s,t}) - \lambda^* \frac{1}{r} + \sum_{(u,v)\in\mathcal{R}} p_{u,v} \\
& \Big(\frac{r}{r + \overline{T}_{s,t}^{-1}} h^*((u,v)(0,i)(0,j)(s,t)) \\
& + \frac{\overline{T}_{s,t}^{-1}}{r + \overline{T}_{s,t}^{-1}} h^*((u,v)(0,i)(0,j)(0,t))\Big), \\
& -\lambda^* \frac{1}{r} + \sum_{(u,v)} p_{u,v} h^*((u,v)(0,i)(0,j)(0,k))\Big\}
\end{aligned}$$

for all $(s,t) \in \mathcal{R}$ and for all $i,\ j,\ k \in \mathcal{D}$.

For the event that two ships are idling when the load (s,t) arrives, i.e., for the state $(s,t)(0,i)(0,j)(m,n)$, the controller has the following three controls:

1. assign the ship at port i to the new load (s,t),
2. assign the ship at port j, and
3. reject the load.

The Bellman equation is

$$
\begin{aligned}
h^*((s,t)(0,i)(0,j)(m,n)) = min\Big\{ & -b\tau_{s,t} + c(\tau_{i,s} + \tau_{s,t}) - \lambda^*\frac{1}{r} + \sum_{(u,v)\in\mathcal{R}} p_{u,v} \\
& \Big(\frac{r}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{s,t}} h^*((u,v)(0,j)(m,n)(s,t)) \\
& + \frac{\overline{T}^{-1}_{m,n}}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{s,t}} h^*((u,v)(0,j)(0,n)(s,t)) \\
& + \frac{\overline{T}^{-1}_{s,t}}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{s,t}} h^*((u,v)(0,j)(0,t)(m,n))\Big), \\
& -b\tau_{s,t} + c(\tau_{j,s} + \tau_{s,t}) - \lambda^*\frac{1}{r} + \sum_{(u,v)\in\mathcal{R}} p_{u,v} \\
& \Big(\frac{r}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{s,t}} h^*((u,v)(0,i)(m,n)(s,t)) \\
& + \frac{\overline{T}^{-1}_{m,n}}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{s,t}} h^*((u,v)(0,i)(0,n)(s,t)) \\
& + \frac{\overline{T}^{-1}_{s,t}}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{s,t}} h^*((u,v)(0,i)(0,t)(m,n))\Big), \\
& -\lambda^*\frac{1}{r} + \sum_{(u,v)} p_{u,v} \\
& \Big(\frac{r}{r + \overline{T}^{-1}_{m,n}} h^*((u,v)(0,i)(0,j)(m,n)) \\
& + \frac{\overline{T}^{-1}_{m,n}}{r + \overline{T}^{-1}_{m,n}} h^*((u,v)(0,i)(0,j)(0,n))\Big)\Big\}
\end{aligned}
$$

for all (s,t), (m,n) in $\mathcal{R}$ and for all i, $j \in \mathcal{D}$.

For the event that only one ship is idling when the load (s, t) arrives, i.e., for the state $(s, t)(0, i)(m, n)(q, r)$, the controller has the following two controls:

1. assign the ship at port i to the new load (s, t), and
2. reject the load.

The Bellman equation is

$$
\begin{aligned}
&h^*((s,t)(0,i)(m,n)(q,r)) \\
&= min\Bigg\{ -b\tau_{s,t} + c(\tau_{i,s}+\tau_{s,t}) - \lambda^*\Big(\frac{1}{r} + \frac{1}{\overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{q,r} + \overline{T}^{-1}_{s,t}}\Big) + \sum_{(u,v)\in\mathcal{R}} p_{u,v} \\
&\quad \Bigg[\frac{\overline{T}^{-1}_{m,n}}{\overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{q,r} + \overline{T}^{-1}_{s,t}}\Big(\frac{r}{r + \overline{T}^{-1}_{q,r} + \overline{T}^{-1}_{s,t}} h^*((u,v)(0,n)(s,t)(q,r)) \\
&\quad + \frac{\overline{T}^{-1}_{q,r}}{r + \overline{T}^{-1}_{q,r} + \overline{T}^{-1}_{s,t}} h^*((u,v)(0,n)(0,r)(s,t)) \\
&\quad + \frac{\overline{T}^{-1}_{s,t}}{r + \overline{T}^{-1}_{q,r} + \overline{T}^{-1}_{s,t}} h^*((u,v)(0,n)(0,t)(q,r))\Big) \\
&\quad + \frac{\overline{T}^{-1}_{q,r}}{\overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{q,r} + \overline{T}^{-1}_{s,t}}\Big(\frac{r}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{s,t}} h^*((u,v)(0,r)(m,n)(s,t)) \\
&\quad + \frac{\overline{T}^{-1}_{m,n}}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{s,t}} h^*((u,v)(0,n)(0,r)(s,t)) \\
&\quad + \frac{\overline{T}^{-1}_{s,t}}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{s,t}} h^*((u,v)(0,r)(0,t)(m,n))\Big) \\
&\quad + \frac{\overline{T}^{-1}_{s,t}}{\overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{q,r} + \overline{T}^{-1}_{s,t}}\Big(\frac{r}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{q,r}} h^*((u,v)(0,t)(m,n)(q,r)) \\
&\quad + \frac{\overline{T}^{-1}_{m,n}}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{q,r}} h^*((u,v)(0,n)(0,t)(q,r)) \\
&\quad + \frac{\overline{T}^{-1}_{q,r}}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{q,r}} h^*((u,v)(0,r)(0,t)(m,n))\Big)\Bigg], \\
&\quad - \lambda^*\frac{1}{r} + \sum_{(u,v)\in\mathcal{R}} p_{u,v} \\
&\quad \Big(\frac{r}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{q,r}} h^*((u,v)(0,i)(m,n)(q,r)) \\
&\quad + \frac{\overline{T}^{-1}_{m,n}}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{q,r}} h^*((u,v)(0,i)(0,n)(q,r)) \\
&\quad + \frac{\overline{T}^{-1}_{q,r}}{r + \overline{T}^{-1}_{m,n} + \overline{T}^{-1}_{q,r}} h^*((u,v)(0,i)(0,r)(m,n))\Big)\Bigg\}
\end{aligned}
$$

for all (s,t), (m,n), (q,r) in $\mathcal{R}$ and for all $i \in \mathcal{D}$.

4.2.4 Limitation of queueing approximation

By approximating the problem as a Semi-Markov problem, and by effectively approximating the market as a queueing system, the formulations do not model the event that all of the three ships are busy when the new load (s,t) arrives, implying that the optimal control for such event is 'reject the load.' Hence, a solution (or stationary policy) of the Bellman equations may not optimally assign loads having pickup deadlines. In the simulations that we present in Section 4.3, we put those loads rejected (because all of the ships are busy at the time of arrival) into a queue and later assign them (according to the corresponding stationary policy) to the ships that become available by the deadlines.

4.2.5 Value iteration

The Bellman equations are solved using relative value iterations. Since the state space $\mathcal{S}$ for our problem forms a single recurrent class, a value iteration is guaranteed to converge (Bertsekas, 2005).

The value iteration is

$$\begin{aligned} h_{k+1}(\boldsymbol{i}) = & \min_{\underline{u} \in U(\boldsymbol{i})} \left[G(\boldsymbol{i}, \underline{u}) + \sum_{\boldsymbol{j}=1}^{\boldsymbol{n}} \underline{p}_{\boldsymbol{ij}}(\underline{u}) h_k(\boldsymbol{j}) \right] \\ & - \min_{\underline{u} \in U(\boldsymbol{s})} \left[G(\boldsymbol{s}, \underline{u}) + \sum_{\boldsymbol{j}=1}^{\boldsymbol{n}} \underline{p}_{\boldsymbol{sj}}(\underline{u}) h_k(\boldsymbol{j}) \right], \quad \boldsymbol{i} = 1, \ldots, \boldsymbol{n} \end{aligned} \tag{4.2.2}$$

where $\boldsymbol{s}$ is some fixed state and the subscript k represents each iteration step.

If the value iteration (4.2.2) converges to some vector h, then we obtain λ^*, the optimal average cost per stage for all initial states, by

$$\lambda^* \overline{\tau}_{\boldsymbol{s}}(\mu(\boldsymbol{s})) = \min_{\underline{u} \in U(\boldsymbol{s})} \left[G(\boldsymbol{s}, \underline{u}) + \sum_{\boldsymbol{j}=1}^{\boldsymbol{n}} \underline{p}_{\boldsymbol{sj}}(\underline{u}) h(\boldsymbol{j}) \right]$$

where $\mu(\boldsymbol{s})$ is the optimal control for the state $\boldsymbol{s}$, i.e.,

$$\mu(\boldsymbol{s}) = \operatorname*{argmin}_{\underline{u} \in U(\boldsymbol{s})} \left[G(\boldsymbol{s}, \underline{u}) + \sum_{j=1}^{n} \underline{p}_{\boldsymbol{sj}}(\underline{u}) h(\boldsymbol{j}) \right].$$

Also, the following equality

$$\lambda^* \overline{\tau}_{\boldsymbol{i}}(\mu(\boldsymbol{i})) + h(\boldsymbol{i}) = \min_{\underline{u} \in U(\boldsymbol{i})} \left[G(\boldsymbol{i}, \underline{u}) + \sum_{j=1}^{n} \underline{p}_{\boldsymbol{ij}}(\underline{u}) h(\boldsymbol{j}) \right],$$

holds for all $\boldsymbol{i} \in \mathcal{S}$ where $\mu(\boldsymbol{i})$ is the optimal control for the state $\boldsymbol{i}$, i.e.,

$$\mu(\boldsymbol{i}) = \operatorname*{argmin}_{\underline{u} \in U(\boldsymbol{i})} \left[G(\boldsymbol{i}, \underline{u}) + \sum_{j=1}^{n} \underline{p}_{\boldsymbol{ij}}(\underline{u}) h(\boldsymbol{j}) \right],$$

and h is an associated differential cost vector.

For our example we use the state $\boldsymbol{s} = (1,3)(0,1)(0,1)(0,1)$ as a special state and $\overline{\tau}_{\boldsymbol{s}}(\underline{u}) = 1/r$ for all $\underline{u} \in U(\boldsymbol{s})$. Hence, we attain the optimal average cost per day by

$$\lambda^* = r \cdot \min_{\underline{u} \in U(\boldsymbol{s})} \left[G(\boldsymbol{s}, \underline{u}) + \sum_{j=1}^{n} \underline{p}_{\boldsymbol{sj}}(\underline{u}) h(\boldsymbol{j}) \right]. \tag{4.2.3}$$

Finally, $h(\boldsymbol{s}) = 0$ by definition.

4.2.6 Stationary policy

By solving the Bellman equations by value iteration, we attain the stationary policy for our example, which looks like Table 4.1. The table contains the optimal controls for the example that 0.06 loads arrive per day along each route (total 0.72 loads per day) while the charter hire is 100 monetary unit per day and the sailing cost is 60. The optimal control '1' in the table means 'assign a ship at port 1' while the control '0' means 'reject.' We apply these stationary policies in the simulations that we present in the following section.

Table 4.1: Stationary policy ($p = 0.06$, $b = 100/day$, $c = 60/day$)

State $\boldsymbol{i}$	Optimal control $\mu(\boldsymbol{i})$	Differential cost $h(\boldsymbol{i})$
(1,3)(0,1)(0,1)(0,1)	1	0
(1,4)(0,1)(0,1)(0,1)	1	-114.245
(1,6)(0,1)(0,1)(0,1)	1	-30.2078
(2,4)(0,1)(0,1)(0,1)	1	38.8654
(4,1)(0,1)(0,1)(0,1)	0	65.9656
(4,3)(0,1)(0,1)(0,1)	0	65.9656
(4,6)(0,1)(0,1)(0,1)	0	65.9656
(5,1)(0,1)(0,1)(0,1)	0	65.9656
(5,4)(0,1)(0,1)(0,1)	0	65.9656
...	...	...

4.3 Simulation

We tested the formulation for our example by simulations and compared the performance of the stationary policy with alternative assignment methods.

4.3.1 Simulation setting

We simulate in a discrete time setting in which time advances in one day intervals. Each simulation runs for 365 days. Loads sailing along each route are generated according to Bernoulli process with the success rate p. The rate is the same for all twelve routes. Arrival of new loads becomes known to the controller at the start of a day. Based on the status (busy or available) and location of ships, the controller assigns the new loads at the start of a day. A load must be picked up within four days; otherwise it is lost forever. The sailing time between ports is deterministic and is two, three or four days. Ships earn charter hire of 100 monetary units per day for a laden voyage (i.e., $b = 100$), the same for all routes. Sailing cost per day (denoted by c) is the same for laden voyages and for ballast voyages. Profit for laden voyages is calculated as charter hire minus sailing cost. If the sailing cost is 80 monetary units per day, for example, the profit for a laden voyage is 20 monetary units per day and the profit margin is 20%.

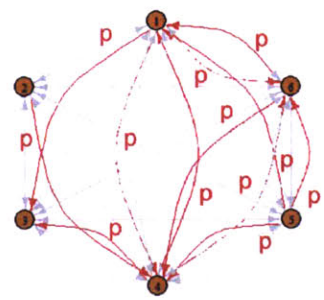


Figure 4-2: Demand generated according to Bernoulli process with rate p

Table 4.2: Queueing analysis

Bernoulli(p)	0.02	0.04	0.06	0.08
Utilization rate(ρ)	0.40	0.62	0.74	0.80
Laden voyage	0.20	0.31	0.36	0.39
Ballast voyage	0.20	0.32	0.38	0.41
Idle time	0.60	0.38	0.27	0.20
P{all ships busy}	0.11	0.31	0.45	0.56

4.3.2 Simulation over different values

We conducted 36 different simulations with varying cargo demand (p) and sailing cost (c). We used Bernoulli rate p of 0.02, 0.04, 0.06, and 0.08. Sailing cost ranged from 10 to 90, incrementing by 10. We repeated each simulation 30 times and calculated the averages for comparison. The Bernoulli rate $p = 0.02$ indicates that the market is slow. In such case, our queueing analysis (an $M/M/3/3$ queue) predicts that each ship is busy 40% of the time and the probability that all three ships are busy is 11% (see Section 3.7). The rate $p = 0.08$ indicates that there is strong demand, in which case each ship is expected to be busy 80% of the time and the probability that all three ships are busy is 56%.

4.3.3 Assignment methods

We applied three different methods, online (or real time), to assign ships during the simulations: greedy, shortest path and stationary policy.

Greedy assignment

This algorithm calculates distances between the loading port of a new load and the location of available ships and assigns the closest available ship to the load. It is the most myopic of the three methods and it mimics common practice in the actual market.

(Deterministic) shortest path algorithm

This method assigns ships to minimize an objective, consisting of the following three costs:

1. Sailing cost: a fixed operating cost per day
2. Opportunity cost: the charter hire of a load when no ship becomes available by the deadline of the load
3. Idling cost: a very small cost (0.01 of sailing cost) imposed on the minimum of either the number of idling ships or the number unassigned loads.

This shortest path algorithm gives the optimal assignment based on known cargoes (so it is less myopic than the greedy assignment). This method is not appropriate for online applications since the state space is very large and a state space search takes a long time. However, it was used in this simulation to provide a benchmark for comparisons.

Stationary policy

This method applies the stationary policy obtained (before simulations) from the Bellman equation. During simulations, it first translates new load and the status (busy or idle) and location of ships into one of our defined states (see Section 4.2.1) and then looks up the stationary policy table (see Table 4.1) for optimal control.

It is noteworthy that by having such stationary policy table before simulations, we can roughly estimate how often this method will reject new loads depending on the profit margin of loads in a market. Figure 4-3 shows the number of states, for

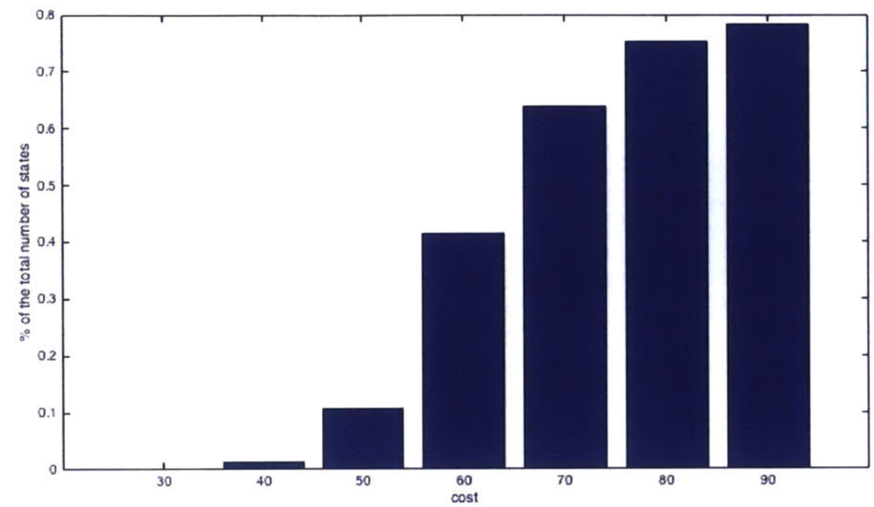


Figure 4-3: Number of rejection states as % of total states

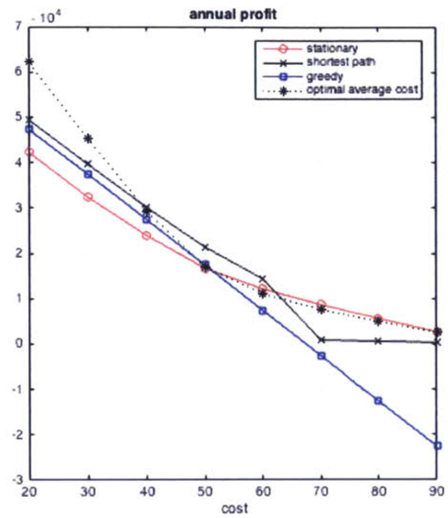
which new load is rejected, as a percentage of the total number of states. When the profit margin is 10% (or $c = 90$), the stationary policy rejects a new load in 78% of the total states. When the profit margin is 30% (or $c = 70$), the stationary policy never rejects a load, implying that the stationary policy becomes the same as the greedy assignment.

It is also notable that the number of the states for which load is rejected in the stationary policy does not change over different values of p (while the profit margin remains unchanged). This suggests that the optimal assignment policy remains valid even when cargo demand fluctuates.

4.3.4 Simulation result

Let us discuss the simulation result for $p = 0.08$ (busy market) (See Figure 4-4). The result suggests that when the sailing cost is between 70 and 90 (profit margin 10-30%) the stationary policy yields the highest profit.

When the sailing cost is 60 (the profit margin 40%), the stationary policy and the shortest path algorithm produce similar profits. However, *the stationary policy yields comparable profit while serving far fewer loads* (45 less). In other words, the stationary policy efficiently maximizes profit while selectively rejecting loads.



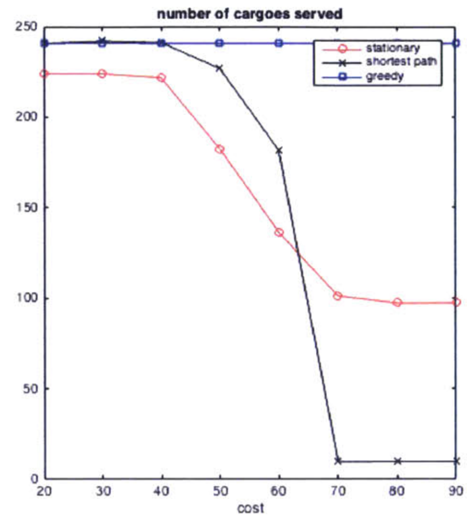


Figure 4-4: Simulation result, $p = 0.08$

While the shortest path algorithm produces the highest profit when the profit margin is high (sailing cost between 20 to 60), its profit drops to almost zero when the sailing cost is above 70. This drop happens because the shortest path algorithm optimizes using only known loads. The algorithm keeps ships idling at port 3 once they discharge at port 3 because any new load will only increase the objective (cost). For example, imagine a new load from port 4 to port 1 which would have a profit of 120 (4 $days \times 30$). If the sailing cost is 70, the ballast trip to move a ship idling at port 3 to port 4 would cost more ($140 = 2\ days \times 70$) than the profit.

Most interestingly, when the margins are very high (60% or above), the greedy method yields comparable profit to the stationary policy and to the shortest path algorithm. This result suggests that *when the overall market is highly profitable, even the most myopic assignment gives good results.* The controller can maximize its profit by accepting any load and assigning the closest available ship. Such result is actually anticipated based on the stationary policy obtained from the Bellman equation. When the margins are high, the optimal stationary policy rejects no loads (see Figure 4-3).

We can observe similar results for less busy markets (or fewer cargo arrivals) while the relative performance of the stationary policy slightly improves. This corresponds to the cases where $p = 0.06$, $p = 0.04$, or $p = 0.02$. It is notable that the stationary

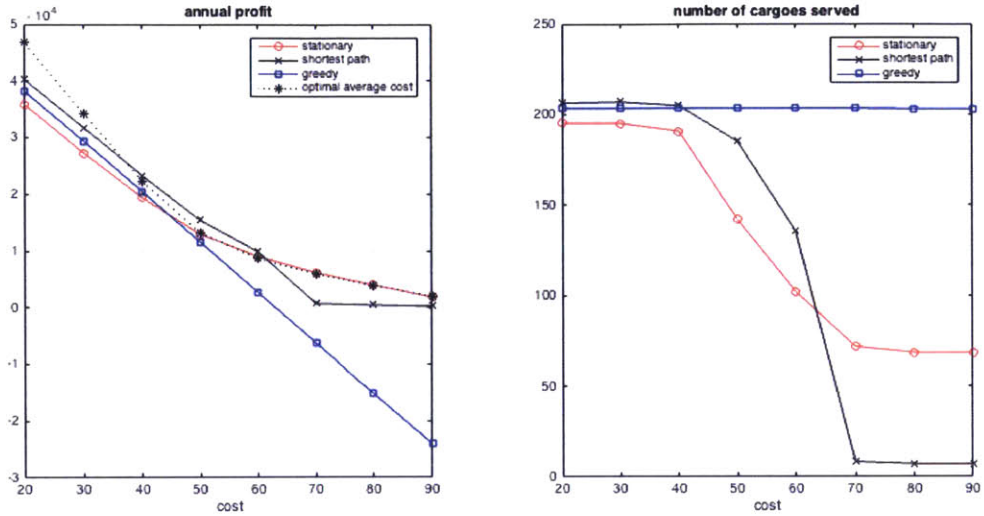


Figure 4-5: Simulation result, $p = 0.06$

policy still yields the highest profit when the profit margins are low while rejecting some loads. This result was also anticipated by observing the stationary policy, which does not depend on the values of p.

The dotted lines in the result figures show the optimal average cost calculated from the value iterations (by (4.2.3)). This *analytical* solution gives good prediction for the simulation performance of the stationary policy, especially for the cases of low profit margins and low p values. Its divergence from the simulation performance for the cases of high profit margins and high p values remains to be investigated although we suspect backlogged loads when all three ships are busy may have played a role.

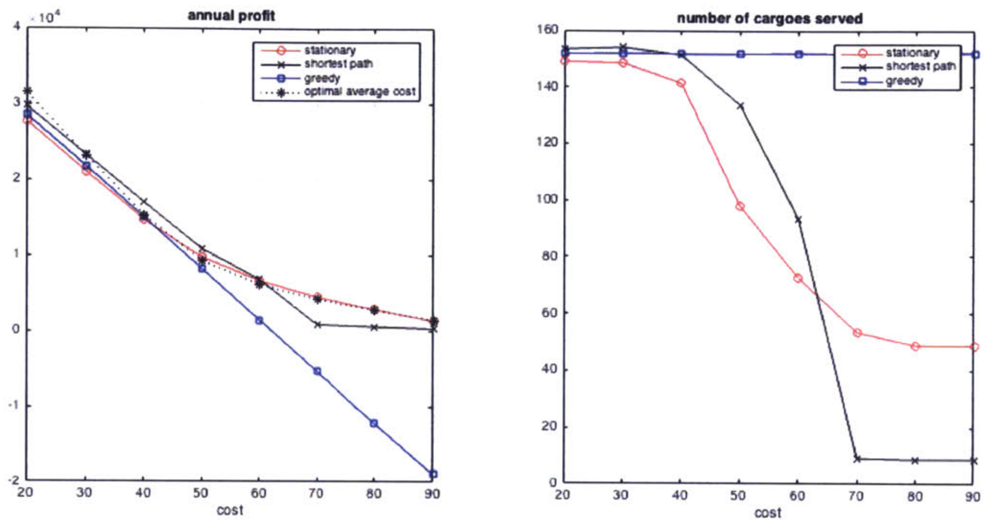


Figure 4-6: Simulation result, $p = 0.04$

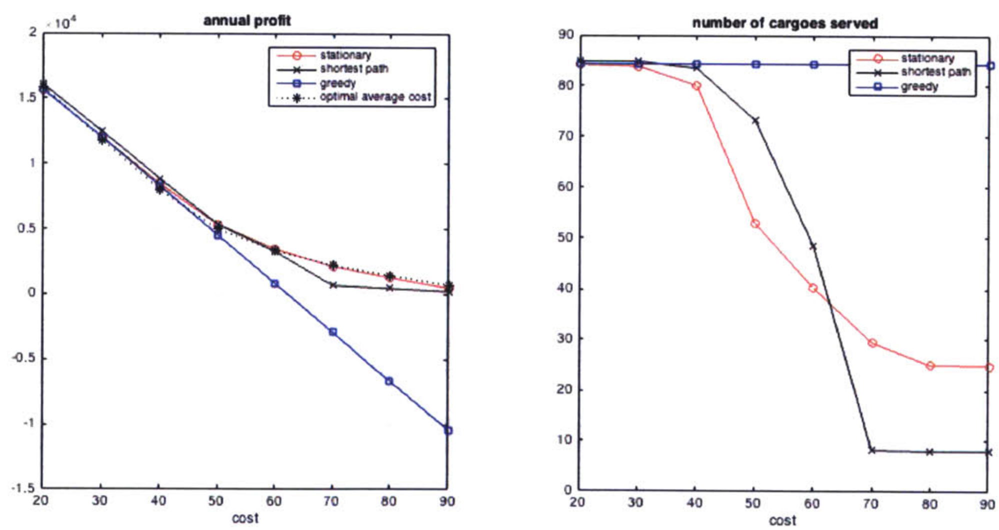


Figure 4-7: Simulation result, $p = 0.02$

Chapter 5

Conclusions and Future Research

We have applied various analytical methods to address the problems of product tanker shipping companies. The basic underlying assumption is to approximate the market as a queueing system, which allows the formulations and analytical solutions presented in this thesis. This chapter summarizes the result and suggests areas for future research.

5.1 Summary

5.1.1 Susceptibility to utilization ratio changes

Our queueing model computes utilization ratios, which effectively model supply and demand of the product tanker market. The advantage of this method is to be able to model complex factors such as ton-mile, analysis of which has not been possible with prevalent methods of merely comparing cargo volume with the number of ships in the market. From the queuing model point of view, a change in ton-mile can be interpreted as a change in the mean service time (or a change in the utilization ratio since $\rho = \lambda \cdot \mathbb{E}[S]/n$), thus our model can directly estimate the impact of change in ton-mile.

Queueing theory reveals that the market is highly susceptible to a change in ton-mile (or utilization ratio) when the utilization ratio is greater than 80%. Figure 5-1

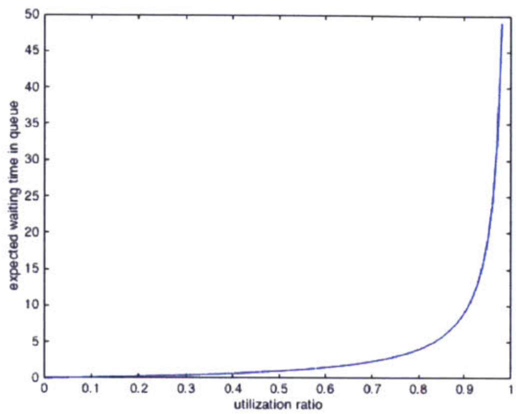


Figure 5-1: Sensitivity to utilization ratio

Figure 5-2: Singapore-Japan route

illustrates this phenomenon. The expected waiting time in a (load) queue grows exponentially as the utilization ratio becomes close to 1 (and becomes infinite at 1). Waiting time represents the delay between the time a load arrives at the market and when it is actually picked up. Hence, a very long waiting time means the market suffers a severe shortage of ships. We can conclude that the cost of product tanker shipping is also susceptible to a change in utilization ratio when this ratio is over 80%. As shown in Figure 5-2, even high-volume routes — such as Singapore-Japan — experience severe fluctuations in price.

5.1.2 Economy of scale

Because regional freight rate (price) arbitrage is endemic in product tanker shipping, product tanker companies continually review distribution of ships over different regions in order to maximize profit. We apply game theory to this regional ship allocation problem assuming that the utilization ratio of a region represents the utility (or payoff or profit) of ships operating in the market.

Our analysis reveals that a big player with a large fleet has advantages over smaller competitors by optimally distributing ships. The Nash equilibrium predicts that when the distribution of the demand changes, small players will distribute their ships according to the new distribution of the demand. In contrast, the big player will react to the change more conservatively than smaller players. In other words, the big player will cause a shortage by not supplying enough ships to regions with high demand. As a result, the big player obtains a greater payoff.

The result helps explain the presence of so called 'pool operators,' who always try to increase their size by pooling more ships from various shipowners.

5.1.3 Optimal controls with rejections

Finally, the queueing approximation allows formulation of the dynamic ship assignment problem as a semi-Markov average cost problem, attaining analytical solutions for this dynamic (real-time) problem, instead of simulation-based solutions. The solution of a semi-Markov problem is called 'stationary policy' in dynamic programming[1] and it contains optimal controls for given states.

When applied in computer simulations, our optimal solutions (or stationary policies) produce more profit per load than other methods especially when the profit margin of each load is low (see Figure 5-3). Such outperformance is delivered by rejecting loads depending on a specific state. In contrast, when the profit margin is high, the myopic greedy method generates comparable results to those optimal stationary policies. The simulation results also show that cargo-rejecting controls remain valid even when demand fluctuates.

5.2 Areas for future research

This is an area that has a lot of potential for future research.

Utilization ratios obtained by the queueing model assume deterministic sailing time between any two ports. Zero loading and discharging times are also assumed. These assumptions are appropriate especially for the analysis of product tanker shipping in which port congestions seldom happen. To extend our queueing analysis to the more general tramp shipping market, we need to adopt queueing network models (instead of a single queueing system of this thesis), which will be suitable especially for the analysis of drybulk shipping in which congestions are endemic.

[1] Oxymoronically called 'stationary' since the solution is time-independent.

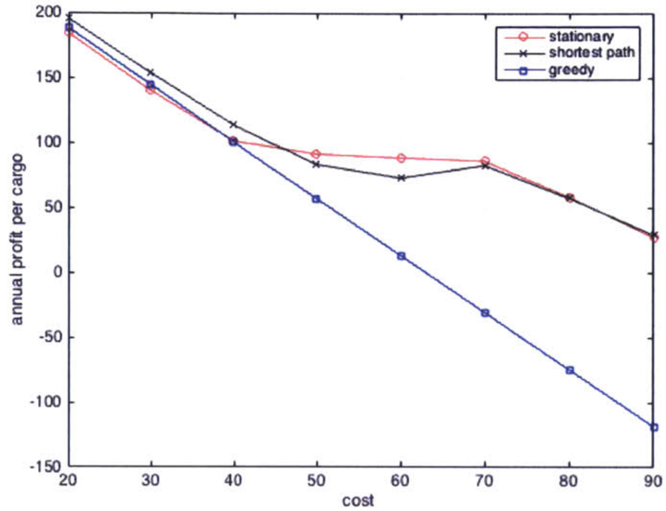


Figure 5-3: Annual profit per cargo, $p = 0.06$

Our formulation and computer simulation show that despite the severe uncertainty in demand, there exist profit-maximizing policies that depend on the profitability of loads. In future research, the performance of these optimal policies under *price uncertainty* needs to be tested. The success of the stationary policies attained by the problem approximation (i.e., semi-Markov formulation) in this thesis, encourages development of simulation-based approximate dynamic programming (ADP) methods. This latest method involves approximation in value space. While the suitability of each of the two approximation methods still needs to be tested, future strategies will require the incorporation of such real-world constraints as heterogeneous fleets.

Finally, it would be very illuminating to model an actual region and empirically test the utilization ratio and dynamic assignment models. The applicability and use of these models in practice are worth further study.

Appendix A

Strategy Iteration

In this game, 100 'small' players who have only one ship each compete with a single 'big' player who has 100 ships. The market consists of two regions, region 1 and region 2. Ships can switch markets for a switching cost (denoted by c).

We introduce the following notation:

$\hat{\rho}^l$	demand-service ratio of region l
$\mathcal{B}$	set of pure strategies of big player ($b_j \in \mathcal{B}$)
b_j	pure strategy of big player, $j = 0, 1, 2, ..., 100$
b_j^l	number of big player's ships in region $l \in \{1, 2\}$ for pure strategy b_j
n_s	total number of small players, i.e., $n_s = 100$
$[n]$	set of small players, i.e., $[n] = \{1, 2, ..., n_s\}$
$[\xi]$	set of pure strategies of each small player, i.e., $[\xi] = \{1, 2\}$
Δ^ξ	set of distributions over $[\xi]$
δ_i	mixed strategy of small player i, i.e., $\delta_i \in \Delta^\xi$
δ_i^l	probability of each small player being in region $l \in \{1, 2\}$
δ_{-i}	mixed strategy of small players except for i
s^l	number of small players' ships in region $l \in \{1, 2\}$
p_s	probability of each small player switching regions
c	switching cost per ship

Small players share the same strategy set $[\xi]$ and, given a strategy of the big player, the utility of small player i choosing region l is a function of the number of small players other than i choosing region l. Hence, conditioned on the strategy of the big player, *small players play a symmetric game*, i.e., $\delta_i = \delta_{-i}$. Since the big player has a finite pure strategy set, a Nash equilibrium exists.

In the initial state, the demand-service ratios for region 1 and region 2 are $\hat{\rho}^1_{(0)} = 90$ and $\hat{\rho}^2_{(0)} = 90$, respectively. In this state, the big player plays the pure strategy $b_{(0)} = (50, 50)$ and each small player (or player i) plays the mixed strategy $\delta_{i(0)} = (0.5, 0.5)$.

In the new state, the demand-service ratios for region 1 and region 2 change to $\hat{\rho}^1 = 120$ and $\hat{\rho}^2 = 60$, respectively. Now we want to find the Nash equilibrium for the new state. We do this by strategy iteration. Each iteration is indexed by (k). The (conditional) utility of the big player when playing $b_{(k)}$ is denoted by $u(b_{(k)}; \cdot)$ and the (conditional) utility of small player i when playing $\delta_{i(k)}$ is denoted by $u(\delta_{i(k)}; \cdot)$.

Given the initial strategy $b_{(0)}$ and the mixed strategy of small players $\delta_{i(k)}$, the utility of the big player is

$$u\big(b_{(k)}; b_{(0)}, \delta_{i(k)}\big) = \frac{\hat{\rho}^1 \cdot b^1_{(k)}}{b^1_{(k)} + s^1_{(k)}} + \frac{\hat{\rho}^2 \cdot b^2_{(k)}}{b^2_{(k)} + s^2_{(k)}} - c \cdot \left| b^1_{(k)} - b^1_{(0)} \right|$$

where $s^1_{(k)}$ and $s^2_{(k)}$ are binomial random variables, i.e., $s^1_{(k)} \sim B[n_s, \delta^1_{i(k)}]$ and $s^2_{(k)} \sim B[n_s, \delta^2_{i(k)}]$. We use notation $B[N, P]$ to refer to the binomial distribution with number of trials in N and probability of success for each trial in P. The probability at the specific value x is denoted by $B[x; N, P]$. Hence, the expected utility of the big player conditioned on the mixed strategy of small players $\delta_{i(k)}$ is

$$\begin{aligned}
\mathbb{E}\Big[u\big(b_{(k)}; b_{(0)}\big)\Big] = & \sum_{s^1_{(k)}=0}^{n_s} \frac{\hat{\rho}_1 \cdot b^1_{(k)}}{b^1_{(k)} + s^1_{(k)}} B\Big[s^1_{(k)}; n_s, \delta^1_{i(k)}\Big] \\
& + \sum_{s^2_{(k)}=0}^{n_s} \frac{\hat{\rho}_2 \cdot b^2_{(k)}}{b^2_{(k)} + s^2_k} B\Big[s^2_{(k)}; n_s, \delta^2_{i(k)}\Big] \\
& - c \cdot \left| b^1_{(k)} - b^1_{(0)} \right|
\end{aligned}$$

With the objective of maximizing its utility, the big player updates its strategy with

$$b_{(k+1)} = \underset{b_{(k)} \in \mathcal{B}}{\operatorname{argmax}} \ \mathbb{E}\Big[u\big(b_{(k)}; b_{(0)}\big)\Big].$$

Now, given the strategy of the big player $b_{(k+1)}$ and the initial state $\delta_{i(0)}$, the utility of small player i becomes

$$\begin{aligned} u\big(\delta_{i(k)}; \delta_{i(0)}, \delta_{-i(k)}, b_{(k+1)}\big) =& \frac{\hat{\rho}^1 \cdot \delta^1_{i(k)}}{b^1_{(k+1)} + s^1_{(k+1)} + 1} \\ &+ \frac{\hat{\rho}^2 \cdot \delta^2_{i(k)}}{b^2_{(k+1)} + s^2_{(k+1)} + 1} \\ &- c \cdot p_s \end{aligned}$$

where $s^1_{(k+1)}$ and $s^2_{(k+1)}$ are the numbers of small players except for i in region 1 and region 2, respectively, given the strategy $b_{(k+1)}$ of the big player. They are binomial random variables, i.e., $s^1_{(k+1)} \sim B\Big[n_s - 1, \delta^1_{-i(k)}\Big]$ and $s^2_{(k+1)} \sim B\Big[n_s - 1, \delta^2_{-i(k)}\Big]$.

Given the mixed strategy $\delta_{i(k)}$ and the initial state $\delta_{i(0)}$, the probability of player i switching regions is

$$p_s = \delta_i(0)^1 \delta_i(k)^2 + \delta_i(0)^2 \delta_i(k)^1 = 0.5(\delta_i(k)^1 + \delta_i(k)^2) = 0.5.$$

With $\delta_i = \delta_{-i}$, due to symmetry, the expected utility of player i is calculated by

$$\begin{aligned} \mathbb{E}\Big[u\big(\delta_{i(k)}; \delta_{i(0)}, \delta_{-i(k)}, b_{(k+1)}\big)\Big] =& \sum_{s^1_{(k+1)}=0}^{n_s-1} \frac{\hat{\rho}^1 \cdot \delta^1_{i(k)}}{b^1_{(k+1)} + s^1_{(k+1)}} B\Big[s^1_{(k+1)}; n_s - 1, \delta^1_{i(k)}\Big] \\ &+ \sum_{s^2_{(k+1)}=0}^{n_s-1} \frac{\hat{\rho}^2 \cdot \delta^2_{i(k)}}{b^2_{(k+1)} + s^2_{(k+1)}} B\Big[s^2_{(k+1)}; n_s - 1, \delta^2_{i(k)}\Big] \\ &- c \cdot p_s. \end{aligned}$$

Finally, small player i updates its strategy with

$$\delta_{i(k+1)} = \underset{\delta_{i(k)} \in \Delta^{\xi}}{\operatorname{argmax}} \ \mathbb{E}\Big[u\big(\delta_{i(k)}; \delta_{i(0)}, \delta_{-i(k)}, b_{(k+1)}\big)\Big],$$

for all $i \in [n]$.

As $k \to \infty$, $\delta_{i(k)}$ and $b_{(k)}$ converge to the Nash equilibrium.

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